Estimation of the Josephson Critical Current of a Single Grain: Percolation Model of the Resistive Transition

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Abstract

The granular HTS is treated here as a S-N mixture. Experimental data are used to determine the percolation threshold f_0 (the volume fraction of superconducting grains at zero resistance) and f_p (corresponding to the appearance of the first spanning superconducting cluster). The latter consists of percolating channels, each carrying the Josephson critical current I_{ch}. We demonstrate that, knowing f_0 and f_p as well as the morphology and orientation of the grains, one can derive realistic estimates of I_{ch}. This is realized by assuming a parallel resistive combination, one resistor being the spanning superconducting cluster, the other the nonspanning network. The former is treated as a percolation problem while the later is described within the effective-medium theory.

Key words: resistive transition, ceramics, Bi(2212), percolation, effective medium approximation, Josephson network

In a S-N mixture we define the "zero-resistance" fraction of the superconducting volume, $f_0 = f(T_0)$, by the condition $\rho(T_0) = 0$, and the percolation threshold f_p that manifests the appearance of the first spanning superconducting cluster, the latter consisting of percolating channels [1]. Let n be their density in the direction of the current. Each channel can carry the Josephson critical current I_{ch} ('ch' means 'channel') without destroying superconductivity in the weak links between grains. I_{ch} depends on the direction of the current and on temperature. The maximal superconducting current that the spanning cluster can withstand is $I_{cl} = nSI_{ch}$, where 'cl' means 'cluster' and S is the cross section of the sample. Obviously nS is the total number of the channels that cross the sample. Zero resistance is achieved at $f = f_0 \equiv f(T_0)$ when the spanning cluster carries the total transport current: $I_{cl}(T_0) = I$. Here f_0 is a function of I and $f_0(I) \geq f_p$, where the equality is reached at very weak transport currents $I \sim I_{ch}$. The latter condition can be hardly realized experimentally, since $I_{ch} \simeq$ 0.1-10 μA, as we show below.

Let us find both f_p and f_0 for our sample. The exact results obtained using the percolation theory (see Refs. [1–4] as reviews) are $f_p^{cont} \approx 0.17$ for continuum and $f_p^{latt} \approx 0.20$ for dense lattices. These results were recently corroborated experimentally [5]. We accept $f_p = f_p^{cont} \approx 0.17$ for our sample. Since $f_p = f(T_p)$, we get $T_p \approx 80$ K.

At $f > f_p \approx 0.17$ the S-N mixture consists of the parallely connected spanning superconducting cluster and the non-spanning network. The voltage across the spanning cluster is zero for currents smaller than I_{cl}. At higher currents the weak links become normal and the voltage drop rises steeply. The non-spanning part has a smooth V-I characteristic. Therefore we can assume that the spanning cluster carries a current that slightly exceeds I_{cl} in order to keep voltages across the two parallel resistors equal. Correspondingly, the non-spanning part carries slightly less than $I - I_{cl}$. In order to find I_{cl}, let us notice that $n \propto \eta^{-2}$, where $\eta \propto (f - f_p)^{-\nu}$ is the correlation length of the percolation problem and $\nu \approx 0.9$ [1]. Thus we get:

$$n_{ab} = \gamma(f - f_p)^{2\nu}/s_{ab}, \qquad (I \parallel ab) \qquad (1)$$

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$$n_c = \gamma (f - f_p)^{2\nu}/s_c, \qquad (I \parallel c) \tag{2}$$

where $\gamma \simeq 1$ and s_{ab}, s_c are the mean cross sections of the grain for $I \parallel ab$ and $I \parallel c$, respectively. Note that $n_{ab}/n_c = m \approx 100$. Zero resistance is achieved at $j = nI_{ch}$. Using this condition and Eqs. (1-2), we find two similar equations:

$$\left(f(T_0^{ab}) - f_p\right)^{2\nu} \approx \frac{I_g^{ab}}{I_{ch}^{ab}(T_0^{ab})}, \tag{3}$$

$$\left(f(T_0^{c}) - f_p\right)^{2\nu} \approx \frac{I_g^{c}}{I_{ch}^{c}(T_0^{c})}, \tag{4}$$

which determine T_0^{ab} and T_0^c, and, in turn, $f_0^{ab} = f(T_0^{ab})$ and $f_0^c = f(T_0^c)$. Here $I_g = js$ is the mean transport current that crosses a single grain, $j = I/S$ is the current density and s stands for either s_{ab} or s_c depending on the current direction. Note that j is constant in our experiment, therefore $I_g^c/I_g^{ab} = m \simeq 100$.

As one can see from Fig. 1, even weak fields $H \leq 750$ Oe affect the resistive transition appreciably. The excess resistivity, $\Delta\rho$, arising due to the magnetic field is apparently almost independent on the direction of the field with respect to the grain orientation or to the direction of the current. This confirms our assertion that the field affects the intergrain point-like weak links and that the resistivity associated with the vortex motion is unimportant in our case.

The resistive transition becomes "field-sensitive" below certain temperature T_H that can be estimated from Fig. 1 as $T_H^{ab} \approx 92$ K for $I \parallel ab$ and $T_H^c \approx 75$ K for $I \parallel c$. Above T_H the resistivity is almost field independent.

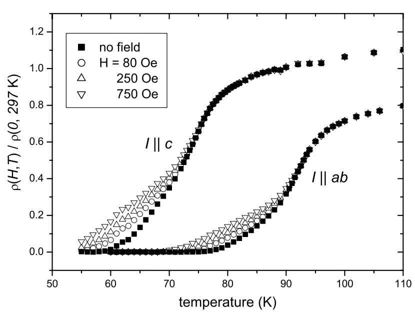

Fig. 1. Resistivities for $I \parallel ab$ (ρ_{ab}) and for $I \parallel c$ (ρ_c), normalized by the corresponding value at $T = 297$ K.

This can be expected, noticing that the Josephson critical current I_{ch} vanishes as $T \to T_{onset}$, whereas I_g, the grain bulk critical current, remains finite, so just below T_{onset} we have $I_{ch} \ll I_g$. In this situation the magnetic field, that further suppresses I_{ch}, has almost no effect on $\rho(T)$. The latter becomes "field-sensitive" provided $I_{ch} \simeq I_g$. Correspondingly, the condition that determines T_H is

$$I_{ch}^{ab}(T_H^{ab}) \simeq I_g^{ab}, \quad I_{ch}^{c}(T_H^{c}) \simeq I_g^{c}. \tag{5}$$

Since $I_g^c/I_g^{ab} = m$ is of the order of the ratio of the crossectional areas of the grains in the corresponding directions (thus for $BSCCO$ one gets $m \approx 100$), it is clear that T_H^c should be considerably less than T_H^{ab}, as indeed observed experimentally. Moreover, using Eq. (5) one can obtain an estimate of I_{ch}, specifically $I_{ch}(92\text{ K}) \approx 0.03$ μA and $I_{ch}(75\text{ K}) \approx 2$ μA. The fact that $T_H^{ab} > T_p$, i.e., the effect of the magnetic field on ρ_{ab} becomes noticeable even in the absence of the spanning cluster, does not contradict our approach. Suppression of I_{ch} results in appearance of additional resistivity in the non-spanning part also, but the quantitative analysis of this effect is quite complex and will be carried out elsewhere.

There are two problems regarding the applicability of the effective medium approximation (EMA) in these ceramics. First, the presence of finite I_{ch} results in additional resistivity not accounted for by EMA. Second, the percolation of current via the spanning cluster breaks the consistency of the EMA and requires consideration of the two-resistor model. Thus under an applied magnetic field of $H = 750$ Oe, we get two competing effects: the appearance of additional resistivity due to suppression of weak links that should worsen the EMA fit, and a suppression of I_{cl} that works in favor of applicability of the EMA. The latter effect proves to be decisive, since the spanning cluster is responsible for the very fast drop of ρ_{ab} at $T < T_p$ in the absence of the field. Therefore it is not surprising that at $H = 750$ Oe we get a consistent EMA picture at all $T > T_H^c \approx 75$ K, whereas at $H = 0$ the applicability of the EMA is restricted by the condition $T > T_p \approx 80$ K.

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