Nuclear magnetic relaxation rate in the vortex state of a chiral p-wave superconductor

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Abstract

The site-selective nuclear spin-lattice relaxation rate T_1^{-1} is theoretically studied inside a vortex core in a chiral p-wave superconductor within the framework of the quasiclassical theory of superconductivity. It is found that T_1^{-1} at the vortex center depends on the sense of the chirality relative to the sense of the magnetic field. Our numerical result shows a characteristic difference in T_1^{-1} between the two chiral states, $\bar{k}_x + i\bar{k}_y$ and $\bar{k}_x - i\bar{k}_y$ under the magnetic field.

Key words: vortex core, NMR, nuclear spin-lattice relaxation rate, chiral p-wave superconductivity

Site-selective nuclear magnetic resonance (NMR) method was recently revealed to be a powerful tool for experimentally investigating the electronic structure inside vortex cores in the mixed state of type-II superconductors (see Refs. [1–3] and references cited therein). The site-selective NMR at the impurity site was also proposed theoretically [4]. The NMR technique as a probe of the electronic structure with spatial resolution is expected to reveal pairing symmetry of unconventional superconductors, because in spatially inhomogeneous systems there appear properties specific to the unconventional superconductivity.

The vortex core is one of such inhomogeneous superconducting systems. In this paper, we study the site-selective nuclear spin-lattice relaxation rate T_1^{-1} inside the vortex core in a chiral p-wave superconductor with an unconventional pairing [5] $\mathbf{d} = \bar{\mathbf{z}}(\bar{k}_x \pm i\bar{k}_y)$. We find that $T_1^{-1}(T)$ exhibits a characteristic chirality dependence as a function of the temperature T, which might be experimentally observed as a sign of the chiral pairing state.

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The system is assumed to be a two dimensional conduction layer perpendicular to the magnetic field applied along the z axis. From now on, the notations are the same as those in Ref. [6]. We consider the quasiclassical Green function

$$\hat{g}(i\omega_n, \mathbf{r}, \bar{\mathbf{k}}) = -i\pi \begin{pmatrix} g & if \\ -if^\dagger & -g \end{pmatrix}, \tag{1}$$

which is the solution of the Eilenberger equation [7],

$$iv_{\mathrm{F}}\bar{\mathbf{k}} \cdot \nabla \hat{g} + \left[i\omega_n \hat{\tau}_3 - \hat{\Delta}, \hat{g} \right] = 0. \tag{2}$$

From the spin-spin correlation function [1], we obtain the expression for T_1^{-1} in terms of $\hat{g}$,

$$\frac{T_1^{-1}(T)}{T_1^{-1}(T_{\mathrm{c}})} = \frac{1}{4T_{\mathrm{c}}} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{\cosh^2(\omega/2T)} W(\omega, -\omega), \tag{3}$$

$$W(\omega, \omega') = \langle a_{11}(\omega) \rangle \langle a_{22}(\omega') \rangle - \langle a_{12}(\omega) \rangle \langle a_{21}(\omega') \rangle, \tag{4}$$

where the spectral function $\hat{a}(\omega, \mathbf{r}, \bar{\mathbf{k}}) = \left(a_{ij} \right)$ is given as

$$\hat{a}(\omega, \mathbf{r}, \bar{\mathbf{k}}) = \frac{-i}{2\pi} \hat{\tau}_3 \Big[\ \hat{g}(i\omega_n \to \omega - i\eta, \mathbf{r}, \bar{\mathbf{k}})$$

$$- \hat{g}(i\omega_n \to \omega + i\eta, \mathbf{r}, \bar{\mathbf{k}}) \Big], \qquad (5)$$

the symbol $\langle \cdots \rangle$ represents the average over the Fermi surface, and η is a small positive constant roughly representing the impurity effect.

Substituting vortex-center solution $\hat{g}$ [6] based on the so-called zero-core vortex model [8] into Eq. (4), we obtain analytical expressions for T_1^{-1} at the vortex center in the case of each chiral state, $\bar{k}_x \pm i\bar{k}_y$.

In Fig. 1(a), we show the result obtained by numerically integrating those expressions with isotropic Fermi surface. It is noticeable that T_1^{-1} of the $\bar{k}_x - i\bar{k}_y$ state is quite different from that of the $\bar{k}_x + i\bar{k}_y$ state. Note that the magnetic field is applied in positive direction of the z axis. In the $\bar{k}_x - i\bar{k}_y$ state, $T_1^{-1}(T)$ is suppressed in wide T region. This is because the second term in Eq. (4) composed of the anomalous Green functions f and $f^\dagger$ (or a_{12} and a_{21}) is nonzero at the vortex center [6] and has minus contribution to T_1^{-1} in the $\bar{k}_x - i\bar{k}_y$ state, while it is zero in the other state.

It was pointed out in Ref. [9] that the impurity scattering rate inside the vortex core of the $\bar{k}_x - i\bar{k}_y$ state was one order smaller than that of the $\bar{k}_x + i\bar{k}_y$ state. In an actual situation, therefore, such T_1^{-1} as plotted in Fig. 1(b) is anticipated. Here, T_1^{-1} is calculated with smaller η for the $\bar{k}_x - i\bar{k}_y$ state. It is noted that T_1^{-1} almost vanishes in the $\bar{k}_x - i\bar{k}_y$ state.

We note that our present result is in contrast to a corresponding theoretical result for $T_1^{-1}(T)$ of Ref. [10] obtained in the quantum limit ($k_{\rm F}\xi \sim 1$). In the result of Ref. [10], there is not such suppression of $T_1^{-1}(T)$ as seen in our Fig. 1. A reasonable origin of this difference is as follows. The calculations of T_1^{-1} in Ref. [10] are in the quantum limit ($k_{\rm F}\xi \sim 1$) where the energy spectrum (the diagonal part of $\hat{a}$) inside the vortex core is quantized and it dominantly determines $T_1^{-1}(T)$ as pointed out in Ref. [10], while we base our calculations on the quasiclassical theory relevant in the opposite limit $k_{\rm F}\xi \gg 1$ where the vortex core spectrum is continuous and the coherence factor in Eq. (4) (especially for the off-diagonal part of $\hat{a}$) determines $T_1^{-1}(T)$.

The chiral superconductivity is expected to be realized in a material Sr_2RuO_4 [5,11]. The result of this paper (Fig. 1), i.e., the chirality dependence of T_1^{-1}, might be observed experimentally by the site-selective NMR method [2,3] in Sr_2RuO_4. and such observations are expected to be helpful to identify the pairing symmetry in this material. Unfortunately, it is certainly difficult to perform such a NMR experiment because the upper critical field is too small to attain the relevant resonance frequency of NMR in the case of applying the magnetic field along the $c(z)$ axis perpendicular to the conduction layers of Sr_2RuO_4. However, the upper critical field is larger in the case of applying the magnetic field along the conduction layers of this material, and therefore the NMR experiments might be

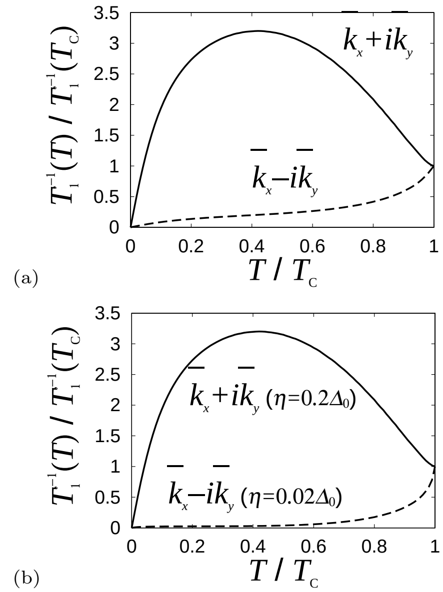

Fig. 1. T_1^{-1} vs T at the vortex center. The magnetic field is applied in positive direction of the z axis. The result for the $\bar{k}_x + i\bar{k}_y$ state is identical to that for the s-wave pairing state. The parameter η represents the smearing effect of the impurities. (a) $\eta = 0.2\Delta_0$ (Δ_0 is the gap amplitude at $T = 0$). (b) η is set as $\eta = 0.02\Delta_0$ only for the $\bar{k}_x - i\bar{k}_y$ state (see text).

possible by applying the magnetic field inclined to the c axis in Sr_2RuO_4. To theoretically predict detailed behavior of T_1^{-1} for the inclined magnetic fields, further analysis will be needed. Such an analysis is left for future studies.

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