The Anderson transition due to random spin-orbit coupling in two-dimension

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Abstract

We report an analysis of the Anderson transition in an SU(2) model with chiral symmetry. Clear single parameter scaling behaviour is observed. We estimate the critical exponent for the divergence of the localization length to be $\nu = 2.72 \pm .02$ indicating that the transition belongs to the symplectic universality class.

Key words: Anderson transition; spin-orbit coupling; chiral symmetry

1. Introduction

It is thought that the critical phenomena of the Anderson transition may be classified according to three universality classes (orthogonal, unitary and symplectic) depending on the symmetries of the Hamiltonian, i.e. time reversal and spin rotation symmetries. A different value of the critical exponent ν describing the divergence of the localization length is expected to characterize each universality class.

In the absence of any diagonal disorder the SU(2) model Hamiltonian may have chiral symmetry. This depends on the boundary conditions and the number of sites in each direction. Near the band center chiral symmetry affects the localization of electrons e.g. an even-odd system size dependence of the quasi-1d localization length is observed [1]. Here we investigate whether or not chiral symmetry also affects the critical exponent ν.

A system belongs to the symplectic universality class if its Hamiltonian commutes with a time reversal operator $\mathcal{T}$ that satisfies $\mathcal{T}^2 = -1$ e.g. systems with significant spin-orbit coupling. Systems in this universality class exhibit an Anderson transition even in 2D. Recently [2] we analysed the SU(2) model with an on-site random potential (diagonal disorder), which breaks chiral symmetry, and estimated the critical exponent $\nu = 2.73 \pm .02$.

Here we study the SU(2) model without an on-site random potential. In spite of the fact that there is no random potential, we find that there is an Anderson transition at a critical energy E_c. Further this energy E_c is far from the the band center and chiral symmetry does not affect the observed critical phenomena.

2. Model and Method

The Hamiltonian of the SU(2) model describes non-interacting electrons on a simple square lattice with nearest neighbour SU(2) random hopping

$$H = -\sum_{\langle i,j\rangle,\sigma,\sigma'} \begin{pmatrix} e^{i\alpha_{ij}}\cos\beta_{ij} & e^{i\gamma_{ij}}\sin\beta_{ij} \\ -e^{-i\gamma_{ij}}\sin\beta_{ij} & e^{-i\alpha_{ij}}\cos\beta_{ij} \end{pmatrix}_{\sigma\sigma'} c^{\dagger}_{i\sigma} c_{j\sigma'} \quad (1)$$

where $c^{\dagger}_{i\sigma}(c_{i\sigma})$ denotes the creation (annihilation) operator of an electron at the site i with spin σ. We distribute hopping matrices randomly and independently with uniform probability on the group SU(2). This corresponds to α and γ uniformly distributed in the range

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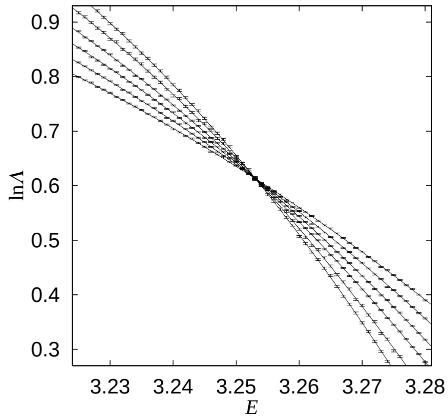

Fig. 1. $\ln \Lambda$ vs Fermi energy E for system sizes $L = 11, 16, 23, 32, 47, 64$. The solid lines are the fit.

$[0, 2\pi)$, and β distributed in the range $[0, \pi/2]$ according to the probability density, $P(\beta)\mathrm{d}\beta = \sin(2\beta)\mathrm{d}\beta$.

We calculate the localization length λ on a quasi-1d strip whose width is L with the transfer matrix method [3] and analyse the dependence of the re-normalized localization length $\Lambda = \lambda/L$ on the energy and the width of the strip [4]. We also calculate the density of states with the Green's function iteration method [5].

In the course of our calculations we noticed a technical problem with the method described in [3]. When fitting the numerical data it is important to have not only an estimate of the localization length λ but also an estimate $\delta\lambda$ of how accurately λ has been estimated. Otherwise a reliable fitting of the data is not possible. We have found that the error $\delta\lambda$ is over estimated by the method of [3]. To overcome this problem we divide the quasi-1d strip into blocks whose lengths are 10^3 and calculate the quasi-1d localization length λ_i(i denotes the block) in each block and accumulate λ_i. ¿From the distribution of $\{\lambda_i\}$ we estimate λ and $\delta\lambda$.

3. Results

We calculated Λ as a function of Fermi energy for systems with widths $L = 11, 16, 23, 32, 47$ and 64. The accuracy of the localization length data is 0.1%, with the exceptions of $L = 47, 64$ where the accuracy is 0.2%. To achieve this accuracy systems of length of the order of 10^7 to 10^8 are required. We impose the periodic boundary conditions in the transverse direction. For this choice of boundary condition systems with even L have chiral symmetry, while those with odd L do not. We fit even and odd data separately first and then in combination. The results are displayed in Table 1. The estimates of the critical parameters for all three fits are in close coincidence and we conclude that chiral symmetry does not change the universality class. In Fig. 1 we show the result for the combined even and odd data. The lines cross at single point, indicating both that corrections to scaling due to irrelevant variables are negligible and also that an even-odd effect does not exist. The estimates ν and $\ln \Lambda_c$ are consistent with those in [2] for the symplectic universality class.

The density of states for a system of the size 100×100 averaged over 10^3 ensembles is shown in Fig.2. In the critical region, the density of states changes slowly. This behaviour is different from that of the random magnetic flux U(1) model in 3D where the density of states changes rapidly in the critical region [6]. This might be the reason why we obtain clearer single parameter scaling here than in a study of the critical phenomena of the U(1) model [7].

Table 1
The number of data N_d and goodness of fit Q. The best fit estimates of the critical energy E_c, $\ln \Lambda_c$ and the critical exponent ν with 95% confidence intervals. There are 7 fitting parameters.

	N_d	Q	E_c	$\ln \Lambda_c$	ν
even	159	0.1	3.2531±.0002	0.613±.002	2.72±.02
odd	164	0.2	3.2531±.0002	0.613±.002	2.72±.03
both	323	0.1	3.2531±.0001	0.613±.001	2.72±.02

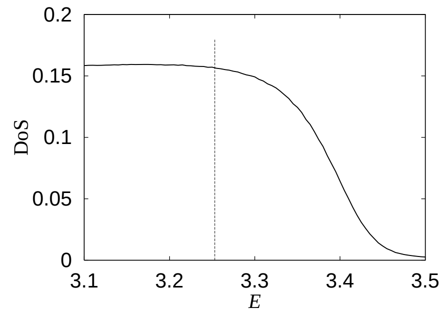

Fig. 2. The DoS near the band edge for a system of size 100×100. The vertical line shows the mobility edge ($E_c \simeq 3.2531$). (The imaginary part of the energy is 0.005.)

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