

# Vibrating wire measurements in dilute $^3\text{He}$ - $^4\text{He}$ solutions at ballistic quasiparticle regime

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## Abstract

We present our vibrating wire resonator data on dilute mixtures. Measurements have been performed for two saturated  $^3\text{He}$  concentrations and pressures ( $x = 6.5\%$ ,  $p = 0$  bar and  $x = 9.5\%$ ,  $p = 10.2$  bar). A DC SQUID readout of the resonator gave a good signal to noise ratio even at low magnetic fields. The data suggests that our experiment has cooled dilute mixtures to the lower temperature than in the earlier experiments has been reached.

*Key words:*  $^3\text{He}$ - $^4\text{He}$  liquid; dilute mixtures; vibrating wire

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In a dilute  $^3\text{He}$ - $^4\text{He}$  solution at temperatures much below the Fermi temperature  $T_F$  of the  $^3\text{He}$  component, the transport properties of the fluid are determined by the  $^3\text{He}$  quasiparticle gas. The quasiparticles form a degenerate Fermi gas in which the mean free path for quasiparticle-quasiparticle scattering varies as  $\lambda \propto T^{-2}$ . At low temperatures  $\lambda$  exceeds the dimensions of the experimental device, which here is the radius  $a$  of the resonator wire. This leads to the transition from the hydrodynamic ( $\lambda \ll a$ ) to the ballistic ( $\lambda \gg a$ ) region as the temperature is varied [1].

The nuclear demagnetization cryostat was equipped with a Lancaster type nested experimental cell which also formed the nuclear cooling stage [2]. The innermost helium volume was completely surrounded by three silver sintered copper plates which formed a minicell [3]. The middle plate had an open volume ( $10 \times 10 \times 1$  mm $^3$ ), where a semicircular vibrating wire resonator (VWR) was placed. In the first cooldown ( $x = 6.5\%$ ,  $p = 0$  bar) we employed only one minicell with the VWR, but later ( $x = 9.5\%$ ,  $p = 10.2$  bar) we had two minicells in use simultaneously, so it was possible to compare the behavior of the VWRs. The resonators were made of tantalum wire ( $a = 62$   $\mu\text{m}$ ,  $\rho_w =$

16700 kg/m $^3$ ). The vacuum frequencies of the VWRs were  $f_0^{vac} = 1202.85$  Hz (VWR1, the 1st cooldown),  $f_0^{vac} = 1202.47$  Hz (VWR1, the 2nd cooldown) and  $f_0^{vac} = 1857.69$  Hz (VWR2, the 2nd cooldown).

The dilution refrigerator was used to precool the nuclear stage in a 9 T magnetic field for about 10 days to reach 6...8 mK. Thereafter the field was slowly reduced to 8 mT and the cell was cooled to the microkelvin region. The final magnetic field provided the field for the VWRs. To span the interesting temperature interval in a reasonable time we had to heat the cell gently (1...10 nW).

The VWRs were driven by a synthesizer by sweeping the frequency over the mechanical resonance at constant excitation current. The induced resonator voltages were amplified by a dc-SQUIDs and monitored by lock-in amplifiers [3]. Very small vibration amplitudes (about 1 nm) of the VWR were used to prevent any significant viscous heating from the wire motion in the fluid [4]. Since the resonances were very broad, the frequency dependent phase and gain of the measurement system had to be corrected for before a Lorentzian line-shape was fitted to the data.

The behavior of the VWR in the fluid is described by the resonance frequency  $f_0$  and the resonance width  $\Delta f$  obtained from Lorentzian fits to the measured data.

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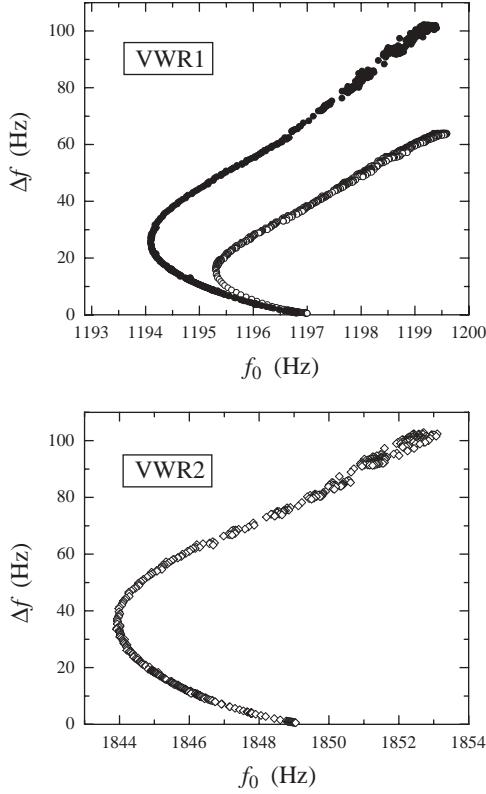


Fig. 1. The vibrating wire resonance width  $\Delta f$  as a function of the resonance frequency  $f_0$  while the temperature is varied. The data for  $x = 9.5\%$ ,  $p = 10.2$  bar was measured with two VWRs ( $\bullet$ ,  $\diamond$ ) and the data for  $x = 6.5\%$ ,  $p = 0$  bar was measured with one VWR ( $\circ$ ). For details, see the text.

The former is sensitive to the hydrodynamic mass of the fluid while the width reflects the damping due to the fluid. In Fig. 1 we have plotted one against the other to recognize the different flow regimes as the temperature and thus the quasiparticle mean free path is varied.

At high temperatures ( $T \gg 2$  mK) the VWR behavior is described by the hydrodynamic treatment ( $\lambda \ll a$ ). The increasing viscosity of the fluid results in an increasing damping as well as an increasing hydrodynamic mass, since the oscillating wire carries along with it a growing shell of fluid determined by the viscous penetration depth ( $\delta \propto \sqrt{\eta}$ ). The data at all concentrations follow practically the same curve (see the VWR1 in Fig. 1). At the highest temperatures the fluid causes almost no damping and the resonance frequency is expected to be  $f_0 = [1 - \rho/(2\rho_w)]f_0^{vac}$ , in which  $\rho$  is the total density of the fluid [5,6]. For the 6.5 % data the extrapolated resonance frequency is about 0.3 Hz below the calculated value (VWR1), but for the 9.5 % data the extrapolated resonances are about 0.5 Hz (VWR1) and 0.9 Hz (VWR2) above the calculated values.

The quasiparticle mean free path depends only

weakly on the  $^3\text{He}$  concentration, so that the transition from the hydrodynamic to the ballistic region occurs at about the same temperature. This is where the curves bend back towards higher resonance frequencies while the damping still increases. At the higher concentration the swing in the plot is larger since the viscosity at the cross-over region is already higher. However, there is a difference between the behavior of two resonators. The curve of the VWR2 turns back towards a higher frequency at about 2 mK compared to about 3 mK of the VWR1. Also the curvature swing is much larger for the VWR2. It is possible that the mean free path effects are due to the closeness of the wall in the case of the VWR1. The data of the VWR2 is more similar to the earlier experiment [1].

In the ballistic regime at the limit of zero temperature ( $\lambda \rightarrow \infty$ ) it is possible to use kinetic arguments to explain the damping of the VWR. In a Fermi gas the frequency width is expected to saturate to a value proportional to the product of the Fermi momentum  $p_F$  and the quasiparticle number density  $n$ , usually written as  $\Delta f(T = 0) = Anp_F/(2\pi^2 a\rho_w) \propto x_3^{4/3}$ . The constant  $A$  of this relation has been argued to lie around  $A = 2.45$ , while the experiments so far have indicated  $A \simeq 2.1$  [1] and our earlier experiments gave  $A \simeq 2.6$  [3]. In the present experiment the two VWRs end up at the same frequency width value at the lowest temperatures. The highest line widths give  $A = 2.60$  and 2.23 for the two concentrations  $x = 6.5\%$  and  $9.5\%$  respectively. The experimental conditions were not ideal for the measurements with the pressurized cell.

As it was noticed some time ago [3], when approaching the zero temperature the resonance frequencies return back to higher values than they have at the high temperature limit. When the damping quasiparticle gas is fully ballistic, the hydrodynamic mass should simply arise from the potential flow of superfluid  $^4\text{He}$  around the wire only, so that the resonance frequencies would be same at the high and low temperature limits. Why this is not so, remains unclear at the moment.

## References

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