Bose-Einstein Condensation and Superfluidity of Dirty Bose Gas

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Abstract

We develop the dilute Bose gas model with random potential in order to understand the Bose system in random media such as ^{4}He in porous glass. Using the random potential taking account of the pore size dependence, we can compare quantitatively the calculated specific heat with the experimental results, without free parameters. The agreement is excellent at low temperatures, which justifies our model. Our model can predict some unobserved phenomena in this system.

Key words: superfluidity; Bose-Einstein Condensation; herium4; random media

1. Introduction

Dirty Bose system is a very important problem for clarifying not only the effect of impurity on the long-range order but also the long-range order itself, for example, the relation between Bose-Einstein condensation and superfluidity. Experimental works have studied this system by liquid ^{4}He in porous Vycor glass, and observed many interesting phenomena[1,2]. Theoretically, some models such as the Bose Hubbard model[3] and the dilute Bose gas model in the delta-functional impurity potential[4], were proposed and gave some qualitative explanations for the experimental results and unobserved predictions. However, there are few works which study this system quantitatively in comparison with the experimental results.

In this work, we improve the dilute Bose gas model with random potential by introducing the pore size dependence of Vycor glass. This model enables us to make quantitative comparison with the experimental results without free parameters, and the agreement of the specific heat and the superfluid density is very good. Furthermore, we can predict some unobserved phenomena, showing the possibility of the observation.

2. The dilute Bose gas model

The Hamiltonian of dilute Bose gas in random potential is given by

$$\begin{aligned}
&\hat{H} - \mu\hat{N} \equiv \hat{K} = \hat{K}_0 + \hat{K}_1 + \hat{K}_2, \\
&\hat{K}_0 = V\Big(-\mu n_0 + \frac{v_0}{2} n_0^2 \Big) + \sum_{\boldsymbol{k}\neq 0} (\varepsilon_{\boldsymbol{k}} - \mu)\hat{a}_{\boldsymbol{k}}^\dagger \hat{a}_{\boldsymbol{k}}, \\
&\hat{K}_1 = \frac{v_0}{2} n_0 \sum_{\boldsymbol{k}\neq 0} \Big(4\hat{a}_{\boldsymbol{k}}^\dagger \hat{a}_{\boldsymbol{k}} + \hat{a}_{\boldsymbol{k}}^\dagger \hat{a}_{-\boldsymbol{k}}^\dagger + \hat{a}_{\boldsymbol{k}} \hat{a}_{-\boldsymbol{k}} \Big), \\
&\hat{K}_2 = \sqrt{\frac{n_0}{V}} \sum_{\boldsymbol{k}\neq 0} \Big(U_{\boldsymbol{k}} \hat{a}_{\boldsymbol{k}}^\dagger + U_{-\boldsymbol{k}} \hat{a}_{\boldsymbol{k}} \Big), \qquad (1)
\end{aligned}$$

where $\hat{a}_{\boldsymbol{k}}^\dagger$ and $\hat{a}_{\boldsymbol{k}}$ are the free particle creation and annihilation operator with the wave number $\boldsymbol{k}$, $U_{\boldsymbol{k}}$ is the external random potential, $\varepsilon_{\boldsymbol{k}} = \hbar^2 k^2/2m$ the kinetic energy of one particle with the mass m, $v_0 = 4\pi a \hbar^2/m$ the coupling constant with the s-wave scattering length a, μ the chemical potential, V the volume of the system and n_0 the number density of condensate particles. The second term $\hat{K}_1$ represents the interaction between two particles, and the third term $\hat{K}_2$ the scattering of one particle by the random potential. Assuming this Hamiltonian, we neglect the higher order terms of noncondensate particle operators. For the random po-

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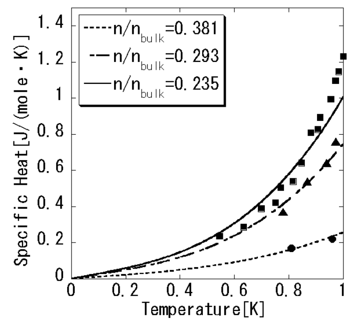
tential $U_{\boldsymbol{k}}$, we take an ensemble-average. Considering the pore size dependence of Vycor glass, we assume the averaged potential

$$\frac{\left\langle U_{\boldsymbol{k}} U_{-\boldsymbol{k}} \right\rangle_{\mathrm{av}}}{V} = R_0 \exp\left[-\frac{k^2}{2k_p^2} \right], \tag{2}$$

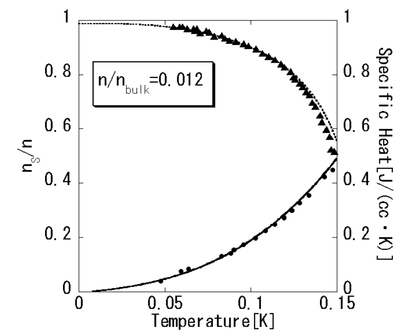
where av denotes the ensemble-average, R_0 is the characteristic strength of the random potential and $k_p = 2\pi/r_p$ is the characteristic wave number of Vycor glass with the average pore size r_p. We carry out the perturbation calculation regarding $\hat{K}_1$ and $\hat{K}_2$ as perturbations, thus obtaining the energy of the system and the condensate density. The superfluid density n_s can be also given by the linear response theory[5].

The density n of the dilute Bose gas ^{4}He inside Vycor glass is estimated from the adsorbed coverage of liquid ^{4}He. Then we can get the critical density corresponding to the critical coverage in which the superfluidity is observed to disappear even near the zero temperature, thus determining R_0 so that the calculated critical density can be consistent with the experimental one. As a result, we can make quantitative comparisons between calculated physical quantities and experimental ones without free parameters.

Shown in Fig. 1 are the data of the specific heat at low temperatures taken from Fig. 1 and Fig. 8 of Reference[1] and our results. Figure 1(a) and (b) show the high and low density data respectively, and the superfluid density is also shown in Fig. 1(b) The good agreements mean our model is quite right at low temperatures. Our model can predict several unobserved effects of the random potential. The most interesting is the reentrant transition(Fig. 2) by which the superfluid density goes to zero again with decreasing the temperature in the low temperature and low density region. We can show that the larger values of R_0, for example, the smaller ratio between the volume of open pores and the total volume of Vycor glass, make the reentrant transition observable.



(a)



(b)

Fig. 1. The specific heat data taken in experiments (plot) and calculations (line). In (a), experimental data are given by Fig.1 of Reference[1], and the circle, triangle and square symbols correspond to full pores, $\sigma = 0.780$ and $\sigma = 0.636$ which are the ratio of the coverage to full pores coverage. In (b), experimental data are given by Fig.8 of Reference[1], and the superfluid density is also compared ($T_c = 0.163$K).

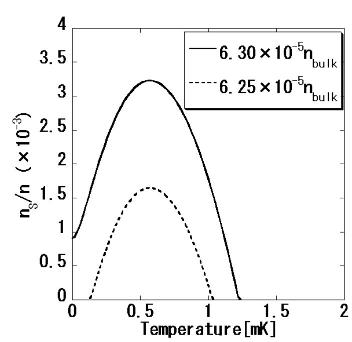


Fig. 2. Dependence of the superfluid density n_s on the temperature in the low temperature and low density region.

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