On Fulde-Ferrel-Larkin-Ovchinnikov phases

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Abstract

We have investigated analytically the nature of the transition to the FFLO superfluid phases in the vicinity of the tricritical point (TCP), where these phases begin to appear. Near the TCP, one can make an expansion of the free energy up to sixth order, both in the order parameter amplitude and wavevector. Restricting ourselves to the order parameter subspace (LO subspace) made of superposition of plane waves of the same wavelength, we obtain first order transitions and a $\cos(\mathbf{q}\cdot\mathbf{r})$ form as the stablest FFLO phase. Moreover, going out of the LO subspace, combining analytical and numerical studies, we show that the actual order parameter at the transition is very close to the simple $\cos(\mathbf{q}\cdot\mathbf{r})$ form.

Key words: FFLO phases ; paramagnetic limit ; nonuniform superfluid state ; cold fermions

1. Introduction

Almost four decades after the theoretical proposition of the Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) phases [1], these phases still raise theoretical and experimental problems. For superconductors in the paramagnetic limit, an external magnetic field induces a difference $2\bar{\mu}$ in the chemical potentials of the two pairing species. For low temperatures, a transition occurs from the normal state to a nonuniform (FFLO) superfluid state. Recently the FFLO state may have been observed in quasi-two-dimensional organic compound [2]. It should also be possible to observe FFLO phases in ultracold fermionic gases where very low temperatures have been obtained recently [3].

The exact nature of the FFLO phases are however still unknown despite recent progress [5,6]. The second order FFLO instability appears below the TCP, $T_{tcp}/T_c \simeq 0.56$, along a line in the $(\bar{\mu}, T)$ plane. It gives rise to a spatial dependence in $\exp(i\mathbf{q}\cdot\mathbf{r})$ of the order parameter but leaves a degeneracy with respect to the direction of $\mathbf{q}$.

Following Houzet et al. [5], we make an expansion of the generalized free energy with respect to the order parameter amplitude and wavevector in the vicinity of the TCP, where the FFLO phases begin to appear. In order to study how the $\mathbf{q}$-orientation degeneracy is lifted, we first restrict ourselves, in section 2, to the order parameter subspace generated by the plane waves $\exp(i\mathbf{q}\cdot\mathbf{r})$. This allows us to compute everything analytically, thus to analyze the reasons that favor one phase compared to another. We find then that the transition to the FFLO phase is first order and that the stablest state is a one-dimensional texture $\Delta(\mathbf{r}) \sim \cos(\mathbf{q}\cdot\mathbf{r})$. In section 3, we show that the actual minimum is only slightly modified when one releases the subspace restriction.

2. The LO subspace

In the LO subspace, the order parameter is written as $\Delta(\mathbf{r}) = \sum_{\mathbf{q_i}} \Delta_i \exp(i\mathbf{q_i}\cdot\mathbf{r})$, with $|\mathbf{q_i}| = q_0$. Since the fourth order term in the order parameter amplitude expansion can be negative, one has to go to sixth order terms whereas fourth order is sufficient for the wavevector expansion. With rescaled variables, the free

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energy reads :

$$F = N_2\bar{\Delta}^2[A_0 - \frac{\bar{q}^2}{3} + \frac{\bar{q}^4}{5}] + N_4\bar{\Delta}^4[\frac{1}{6}(\beta+2)q_0^2 - \frac{1}{4}] + \frac{1}{8}N_6\bar{\Delta}^6 \quad (1)$$

where $A_0(\bar{\mu}, T)$ is a measure of the distance to the spinodal line. $N_p\bar{\Delta}^p = \int |\Delta(\mathbf{r})|^p$ and N_2 is simply the number of plane waves N in $\Delta(\mathbf{r})$. β is defined by $2\beta q_0^2 N_4\bar{\Delta}^4 = -\int \Delta^2(\nabla\Delta^*)^2 + c.c.$. As we will see below, the $\bar{\Delta}^4$ term in (1) can be negative leading to a first order transition. The stablest state minimizing F requires therefore β to be minimum. Making use of :

$$\int [\bar{\Delta}^2(\nabla\bar{\Delta}^*)^2 - |\bar{\Delta}|^2|\nabla\bar{\Delta}|^2] + c.c. = \int [\bar{\Delta}\nabla\bar{\Delta}^* - c.c.]^2 \leq 0$$

we can show that $\beta \geq -\frac{1}{3}$ with the equality $\beta = -\frac{1}{3}$ for *any* real order parameter. As a consequence, the stablest state has a real order parameter. This point can be made clearer when one defines γ by $\gamma \int d\mathbf{r}|\bar{\Delta}|^4 = \bar{q}_0^{-2}\int d\mathbf{r}[\nabla|\bar{\Delta}|^2]^2 = \sum_{\mathbf{q}_i}(\hat{\mathbf{q}}_1 - \hat{\mathbf{q}}_2)^2\bar{\Delta}_{\mathbf{q}_1}\bar{\Delta}^*_{\mathbf{q}_2}\bar{\Delta}_{\mathbf{q}_3}\bar{\Delta}^*_{\mathbf{q}_4}$ where the sum is over $\{\mathbf{q_i}\}$ such that $\mathbf{q_1} + \mathbf{q_3} = \mathbf{q_2} + \mathbf{q_4}$. β and γ can be shown to satisfy $\gamma = 1 - \beta$ thus we want γ to be maximized. By looking at the first expression defining γ, it appears that one needs strong spatial variation for the order parameter. This can be achieved if $\Delta(\mathbf{r})$ has many nodes which favors a real order parameter. The second expression defining γ shows it is favorable to take opposite vectors such that $(\hat{\mathbf{q}}_1 - \hat{\mathbf{q}}_2)^2$ takes its maximum value. Sum of opposite vectors give $\cos(\mathbf{q_0}.\mathbf{r})$ which is a real contribution to the order parameter. We are thus led to consider a real order parameter ($\beta = -\frac{1}{3}$) for which the $\{\mathbf{q_i}\}$ go by pairs $\{\mathbf{q_i}, -\mathbf{q_i}\}$ in the LO subspace. Minimizing the free energy F with respect to q_0^2 gives us $q_0^2 = \frac{5}{6} - \frac{25N_2}{36N_4}\bar{\Delta}^2$ which implies :

$$\frac{F}{\bar{\Delta}^2} = N_2(A_0 - \frac{5}{36}) - \frac{N_4}{54}\bar{\Delta}^2 + \bar{\Delta}^4(\frac{N_6}{8} - \frac{125}{1296}\frac{N_4^2}{N_2}) \quad (2)$$

The second order FFLO transition simply corresponds to $A_0 = 5/36$. We also find a first order transition for any real order parameter in our LO subspace. These first order transition lines are given by :

$$A_0 = \frac{5}{36} + \frac{1}{1458}\,\frac{1}{\frac{N_2N_6}{N_4^2} - \frac{125}{162}} \quad (3)$$

The highest critical temperature is given by the maximum A_0 hence we minimize N_2N_6/N_4^2 with respect to the amplitudes Δ_i. We find it is favorable to decrease the number of plane waves N and the maximum A_0 is given by a simple pair, i.e., $\Delta(\mathbf{r}) \sim \cos(\mathbf{q}\cdot\mathbf{r})$. In that case $N_2N_6/N_4^2 = 10/9$ and then $A_0 = 5/36 + 2.02 10^{-3}$.

3. Actual minimum

We now release our restriction of being in the LO subspace but we still suppose that the order parameter has a one-dimensional form. The free energy, as well as A_0, can still be written in the vicinity of the tricritical point as a universal function of the order parameter [7] and the functional derivative of these expressions give us an ordinary nonlinear differential equation (ONLDE) for the order parameter. Clearly our former simple solution $\cos(\mathbf{q}\cdot\mathbf{r})$ cannot be a solution of this ONLDE. We made an exploration of the solutions of this ONLDE : two spatial frequencies q_0 and q_1, close to the standard FFLO frequency, and their odd harmonic combinations appeared. However, starting with a single frequency, we find that it is not favorable to have a frequency splitting. Therefore we have for the actual minimum a single frequency q_0 and its odd harmonics. Numerically : $\Delta \sim \cos(\mathbf{q}\cdot\mathbf{r}) - 1.33\,10^{-2}\cos(3\,\mathbf{q}\cdot\mathbf{r})$ and $A_0 = 5/36 + 2.7\,10^{-3}$ which is not a significant increase compared to the FFLO result but which is somewhat different from our LO subspace result.

In conclusion, we have treated analytically the FFLO phases in the vicinity of the TCP within a natural subspace generated by plane waves of same wavelength. We have found a first order transition to a planar $\cos(\mathbf{q}\cdot\mathbf{r})$ solution only slightly modified when the subspace restriction is released.

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