Experimental evidence for the weak turbulence on the surface of liquid hydrogen

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Abstract

We present new results of experimental investigations of turbulence of capillary waves on the charged surface of liquid hydrogen. The high-frequency boundary of the inertial interval has been observed for the first time. The boundary frequency ω_b is shifted towards higher frequencies with increasing the wave amplitude η_p at a pumping frequency ω_p in accordance with the theory of weak wave turbulence, $\omega_b \sim \eta_p^{4/3}\omega_p^{23/9}$.

Key words: liquid hydrogen; capillary waves; turbulence

System of nonlinear interacting waves at the surface of a fluid can be described by the kinetic equation analogous to the Boltzmann equation in gas dynamics [1]. In a system of capillary waves there is a frequency interval (inertial interval) which is bounded from below by the pump frequency ω_p and from above by the frequency ω_b at which the viscous damping time τ_v is comparable, by an order of magnitude, to the nonlinear interaction time τ_n: $\tau_n \sim \tau_v$.

The dispersion law for capillary waves $\omega_k^2 = \sigma/\rho k^3$ (ω_k is frequency, k is the wave vector, σ is the surface tension, and ρ is density) is of the decay type, and, hence, the main contribution to the wave interaction comes from the three-wave processes of wave decay into two waves with the conservation of the total wave vector and frequency, as well as from the reverse processes of two-wave confluence into a single wave. Therefore, a constant energy flux to higher frequencies is established in the capillary-wave turbulence regime; hence, it occurs at frequencies higher than the pump frequency (direct cascade).

The viscous damping time of capillary waves decreases with an increasing wave vector as

$$\tau_v^{-1} = 2\nu k^2, \tag{1}$$

where ν is the kinematic viscosity of the fluid.

The nonlinear interaction characteristic time τ_n, is determined by the fluid parameters and the capillary-wave distribution $n(k)$ over the wave vector,

$$\tau_n^{-1} \sim |V_k|^2 n(k) k^2 \omega_k^{-1}, \tag{2}$$

where $V_k \sim (\sigma/\rho^3)^{1/4} k^{9/4}$ is the three-wave nonlinear interaction constant.

The steady-state distribution of surface waves in the inertial interval can be described by the Fourier transform of the pair correlation function $I_k = \langle|\eta_k|^2\rangle$ for the surface deviations $\eta(r,t)$ from the flat state. The correlation function is related to the distribution function $n(k)$ by the expression

$$I_k = n(k)\rho\omega_k/(\sigma k), \tag{3}$$

Equations (1)-(3) can be used to find the wave frequency ω_b (boundary frequency)

$$\omega_b \sim \eta_p^{\beta}\omega_p^{\gamma}, \tag{4}$$

The exponents β and γ are determined by the distribution function $n(k) \sim (k/k_p)^{\alpha}$. For the excitation of surface oscillations in a broad frequency range, the exponent α of the distribution function is -19/4 [2], $\beta = 2.4$, and $\gamma = 19/5$. For the excitation of surface oscil-

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lations by a spectrally narrow pumping, $\alpha = -23/4$, β decreases to $4/3$, and $\gamma = 23/9$.

Experiments were carried out in an optical cell placed in a helium cryostat. A flat capacitor was mounted horizontally inside the cell. Hydrogen was condenser into a container formed by the lower plate and a guard ring 25 mm in diameter and 3 mm in height. The liquid layer was 3 mm thick. The upper capacitor plate was situated at a distance of 4 mm above the liquid surface.

The free fluid surface was charged with the help of radioactive platelet placed on the lower plate of the capacitor. The surface oscillations of liquid hydrogen (standing waves) were excited at one of the resonance frequencies using an ac voltage applied to the guard ring additionally to the dc voltage. Surface oscillations of liquid hydrogen were detected by a laser beam reflected from the surface. The reflected beam was focused by a lens onto a photodetector. The voltage at the photodetector was directly proportional to the beam power $P(t)$. The frequency spectrum P_ω of the total reflected power was obtained by the temporal Fourier transform of the recorded $P(t)$ dependence. In our experiments the squared amplitude of Fourier transform P_ω^2 of the measured signal is directly proportional to the correlation function in the frequency representation; i.e., $I_\omega \propto P_\omega^2$

Fig. 1 demonstrates frequency dependence for P_ω^2 measured at the surface excitation frequency $\omega = 137$ Hz. The wave amplitude at the pump frequency is equal to 0.016 ± 0.009 mm. The frequency at which the P_ω^2 function sharply changes (inertial interval boundary) are indicated by the arrows. This frequency undergoes a high-frequency shift as the wave amplitude increases.

Over a wide frequency range, the P_ω^2 dependence shows a power-law behavior with exponent $m = -3.7 \pm 0.3$. This exponent is close to the estimate obtained in the theoretical work [2] for the case of spectrally narrow pumping.

The boundary frequency of the inertial interval edge

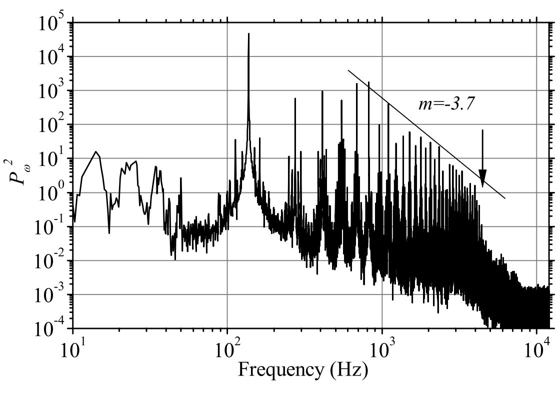

Fig. 1. The P_ω^2 distribution at a pump frequency of 137 Hz.

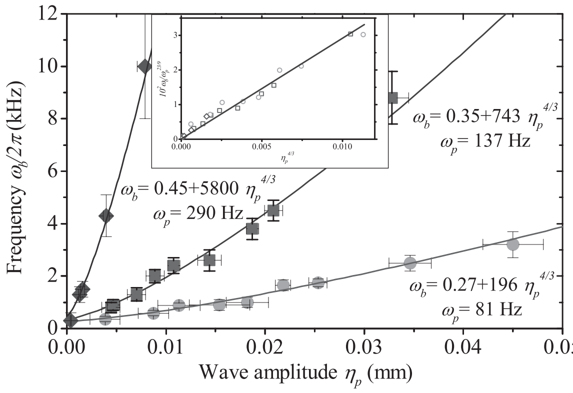

Fig. 2. Boundary frequency as a function of wave amplitude at pump frequency.

is shown in Fig. 2 as a function of wave amplitude for three pump frequencies, 83Hz, 137 Hz and 290 Hz. The solid lines in Fig. 2 correspond to the power-law dependence of the boundary frequency ω_b on the amplitude η with an exponent of 4/3. One can see that the agreement between the experimental points and the theoretical dependence is quite satisfactory.

The dependence of the boundary frequency ω_b on the amplitude in Eq. (4) implies the presence of a scaling law with respect to the pump frequency. One can readily see that, irrespective of the pump frequency ω_p all experimental points ω_b fall on a single straight line in the $\eta^{4/3}$, $\omega_b/\omega_p^{23/9}$ coordinates. The experimental data constructed with these coordinates are shown in insert in Fig. 2. One can state that the experimental points fall on a straight line for three pump frequencies. This confirms the validity of Eq. (4) as well as the results of calculating the capillary-wave distribution function for narrow-band pumping.

Thus, we have demonstrated experimentally that the boundary frequency of the inertial interval is extended to higher frequencies as the wave amplitude at pump frequency rises. This dependence is well described by the power-law behavior with an exponent of 4/3. The experimental results are in qualitative agreement with the theory.

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References

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