Test of a New Field-Theoretical Crossover Equation-of-State

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Abstract

A new field-theoretical crossover equation-of-state model provides a bridge between the asymptotic behavior close to a liquid-gas critical point and the expected mean field behavior farther away. The crossover is based on the beta function for the renormalized fourth-order coupling constant and incorporates the correct asymptotic, crossover, and mean field exponents. Experimental measurements of the isothermal susceptibility, coexistence curve, and heat capacity at constant volume near the ^{3}He critical point compare well with the predictions of this model.

Key words: Equation-of-State; Critical Phenomena; Liquid-Gas Critical Point; ^{3}He Thermodynamic Properties

1. Introduction and theoretical description

In recent years, there has been considerable effort in developing more precise crossover models to describe thermodynamic behavior near a liquid-gas critical point [1]. We have developed a new field-theoretical parametric crossover model (PCM). In this paper, we give a description of this PCM approach and provide an initial experimental test of its predictions.

The basis for the crossover is the equation for the renormalized fourth-order coupling constant in the Ginzburg-Landau-Wilson model of the critical point of $O(1)$ universality systems. This coupling constant, $u(\ell)$ obeys the Renormalization Group equation

$$\frac{du(\ell)}{d\ell} = \beta(u(\ell)) \,. \tag{1}$$

The zero of the beta function, $\beta(u(\ell))$, at the non-zero argument, $u(\ell) = u^*$, controls the critical behavior.

This new model is based on a modification of the parametric formulation of the critical point equation-of-state [2]. In this formulation, the parametric "radial" variables depend on the parameter ℓ through $u(\ell)$. The ℓ-dependence of these variables interpolates between the mean field limit when $u(\ell)$ is very small and the limit associated with asymptotic critical properties when it approaches u^*. Similarly, the parametric "angular" dependence of quantities is described by the mean field equation-of-state when $u(\ell) \ll u^*$ and asymptotes to the most accurate field theoretical predictions of Guida and Zinn-Justin [3] as $u(\ell)$ approaches the stable fixed point of Eq. (1) at $u(\ell) = u^*$.

The equations for the reduced temperature, t, order parameter, m, and ordering field, h, that encode the critical point and crossover behavior are:

$$t = \tau(\ell)^{-1}\left(1-\theta^2\right), \tag{2}$$

$$m = \mu(\ell)^{-1}\left(\frac{u(\ell)}{u^*}\right)^{-1/2}\theta, \tag{3}$$

$$h = \mu(\ell)e^{-\ell d}\left(\frac{u(\ell)}{u^*}\right)^{-1/2}\left[\theta\left(1-\frac{2}{3}\theta^2\right) + \frac{u(\ell)}{u^*}\left(-0.09481\theta^3 + 7.74\times10^{-3}\theta^5\right)\right]. \tag{4}$$

The prefactors $\tau(\ell)$ and $\mu(\ell)$ satisfy the equations

$$\frac{d\ln(\tau(\ell))}{d\ell} = \left(\frac{1}{\nu} + \frac{2-1/\nu}{u^*}\left(u^*-u(\ell)\right)\right) \tag{5}$$

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$$\frac{d\ln(\mu(\ell))}{d\ell} = \left(\frac{d-2}{2} + \frac{1}{2}\left(\eta\left(\frac{u(\ell)}{u^*}\right)^2\right)\right), \qquad (6)$$

where d is the system dimensionality, and ν and η are critical exponents for the correlation length and correlation decay at criticality, respectively. Equations (1)–(6) guarantee proper asymptotic behavior in both the mean field regime and at criticality. Furthermore, the crossover exponents and leading amplitude ratios are also correct in the mean field and asymptotic limits. The precise form of the crossover between mean field and critical behavior is subject to fine-tuning that preserves leading order results. The equations above represent the simplest version of the parametric crossover equation-of-state. It is anticipated that improvements will not lead to substantial quantitative changes in the model predictions.

2. Experimental verification

This new PCM approach was tested using dimensionless isothermal susceptibility, χ_T^*, coexistence curve, $\Delta\rho_{lg}$, and heat capacity at constant volume, C_V^*, measurements near the ^{3}He liquid-gas critical point. These quantities are expected to diverge with a power-law behavior in the asymptotic region close to the transition. Correction-to-scaling terms become important farther away from the transition.

The comparison between theory and experiment was made more sensitive by plotting the susceptibility and coexistence curve data normalized by the leading power law behavior. The initial conditions at $\ell = 1$ for Eqs.(1) and (5), u/u^* and τ, are correlated fundamental model parameters and must be the same for fitting all ^{3}He thermodynamic quantities. We fixed $\tau = 1.0$ in the present analysis. For a particular experiment, the remaining model amplitude parameter adjusts the vertical scale to overlay theory with experiment.

The susceptibility data [4] were analyzed first by manually adjusting the model parameters u/u^* and A_s. The solid lines in Fig. 1a are the theoretical predictions. There is a good fit between theory and experiment over the entire range of experimental measurements. The asymptotic critical amplitudes, $\Gamma_0^{\pm}$, above and below the transition, can be obtained from the fit.

A fit to the shape of the coexistence curve [5] is shown in Fig. 1b, holding u/u^* fixed at the value obtained in the susceptibility fit and adjusting only the amplitude parameter, A_c. The theoretical prediction (solid line) provides a reasonable fit to the coexistence curve measurements for $\mid T/T_c - 1 \mid \leq 10^{-1}$ yielding the critical amplitude B_0. Figure 1c shows a log-log plot of ^{3}He heat capacity measurements [6]. The solid curves are the theoretical prediction. The amplitude A_h was calculated from the universal ratio $R_c = \alpha A_0^+ \Gamma_0^+ / B_0^2$ using already obtained Γ_0^+ and B_0. Temperature dependent analytic background terms are required to fit the heat capacity and coexistence curve data farther away.

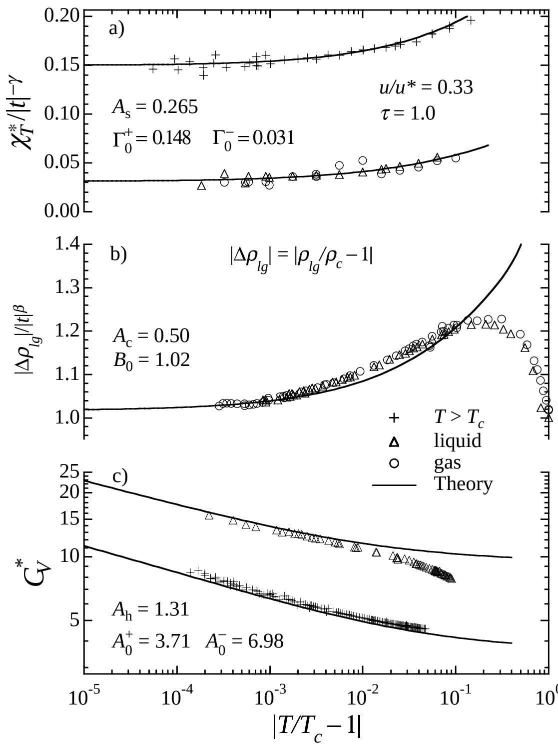

Fig. 1. PCM fit to ^{3}He a) susceptibility, b) coexistence curve, and c) constant-volume heat capacity.

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