Theory of surface plasmons in layered conducting crystals

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Abstract

An integral dispersion equation is derived and used to investigate the spectral and relaxation properties of surface plasma oscillations in layered conductors with quasi-2D motion of charge carriers. The results demonstrate significant differences from the case of an isotropic metal.

Key words: Surface plasmons; Kinetic description; Quasi-2D electron energy spectrum.

1. The standard hydrodynamic theory of surface plasma oscillations is actually based on the isotropic model of metal [1-3]. At the present time there is considerable research attention to synthetic metal-like compounds, having a pronounced layered crystal structure and a sharp anisotropy of both the static and HF electric conductivity. As a rule, such objects are characterized by an effectively reduced dimensionality of electron energy spectrum, i.e., by open Fermi surfaces of the "corrugated cylinder" type [4]. The influence of such anisotropy on the properties of surface plasmons can be taken into account in a consistent way only within the framework of a microscopic description based on the kinetic equation for non-equilibrium electron distribution function ψ .

2. Let the wave vector $\mathbf{k}$ and inner normal $\mathbf{n}$ to the sample surface be directed along principal (assumed to be mutually perpendicular) crystallographic axes; electric field $\mathbf{E}$ lies in the $(\mathbf{k},\mathbf{n})$ plane (TM-wave). If we neglect the retardation due to finite speed of light ($k >> \omega/c$), the basic equations in semi-infinite metal ($x_n \geq 0$) are

$$\nabla \cdot \mathbf{E} = -4\pi e \langle\psi\rangle \equiv -4\pi e \frac{2}{(2\pi\hbar)^3} \int \psi v^{-1} dS_F \qquad (1)$$

$$v_n \partial\psi/\partial x_n + i(\mathbf{k}\cdot\mathbf{v} - \omega - i/\tau)\psi = -e\mathbf{E}\cdot\mathbf{v} \qquad (2)$$

($\mathbf{v} = \partial\varepsilon/\partial\mathbf{p}$ is the electron velocity on Fermi surface S_F ; τ, the electron relaxation time, is expected to satisfy $\omega\tau >> 1$ at low temperatures). Here we assume the simplest (acceptable for ideal surface) "specular" boundary condition: $\psi(+0, v_n) = \psi(+0, -v_n)$. It permits, after even continuation of electric scalar potential to $x_n < 0$, the direct Fourier transformation ($x_n \to q$) of Eqs.(1), (2). In q-representation the solution (of unit value $E_n(0)$) is

$$\mathbf{E}(k,q) = 2\mathbf{Q} e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}/iQ^2\epsilon(\mathbf{Q},\omega) \qquad (3)$$

$$\epsilon \equiv 1 + 4\pi e^2 Q^{-2} \left\langle \frac{\mathbf{Q}\cdot\mathbf{v}}{\mathbf{Q}\cdot\mathbf{v} - \omega} \right\rangle \qquad (4)$$

where $\epsilon(\mathbf{Q},\omega)$ is the dielectric function in boundless metal for longitudinal wave with wave vector $\mathbf{Q} = \mathbf{k} + \mathbf{n}q$; ω means $\omega + i0$ in Eq.(4).

In microscopic consideration the both components of electric field have to be continuous on metal border. Matching the inverse Fourier transform of (3) with the solution in vacant half-space, $\mathbf{E} = (i, 1)\exp(i\mathbf{k}\cdot\mathbf{x} + kx_n - i\omega t)$, we obtain the integral dispersion equation

$$1 + \frac{k}{\pi}\int_{-\infty}^{+\infty} \frac{dq}{Q^2\epsilon(\mathbf{Q},\omega)} = 0. \qquad (5)$$

In principle it solves the stated problem, enabling one to find the spectrum and collisionless damping of surface waves for any specified spectrum of the charge carriers.

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3. We use here the simple, but characteristic, analytical model of a quasi-2D electron energy spectrum in layered conductors [5]:

$$\varepsilon(\mathbf{p}) = (p_x^2 + p_y^2)/2m - (\hbar v_{z0}/a)\cos(ap_z/\hbar). \quad (6)$$

Here a is the lattice period across the layers (the $0Z$ axis) and v_{z0} , the maximum z-projection of electron velocity, is assumed to be small compared to $v_F \equiv \sqrt{2\varepsilon_F/m}$. So the Fermi surface is a slightly corrugated open cylinder, and parameter

$$\mu = \left\langle v_z^2 \right\rangle / \left\langle v_x^2 \right\rangle = (v_{z0}/v_F)^2 << 1 \quad (7)$$

characterizes the anisotropy of local values of high-frequency conductivity. The square of the plasma frequency for bulk waves propagating along $0Z$ is reduced in this same ratio:

$$\begin{aligned} \Omega_\perp^2 &= 4\pi e^2 \left\langle v_x^2 \right\rangle = 4\pi N e^2/m \equiv \Omega^2, \\ \Omega_z^2 &= 4\pi e^2 \left\langle v_z^2 \right\rangle = \mu\Omega^2 \end{aligned} \quad (8)$$

(N is the concentration of free carriers). In this model the longitudinal dielectric functions for the principal directions of the wave vector $\mathbf{K}$, i.e., parallel and perpendicular to $0Z$, are easily calculated to be

$$\begin{aligned} \epsilon_z(K,\omega) &= 1 + 2(\Omega/Kv_F)^2 \left(1 - \omega/\sqrt{\omega^2 - K^2 v_{z0}^2}\right) \\ \epsilon_\perp(K,\omega) &\cong 1 + 2(\Omega/Kv_F)^2 \left(1 - \omega/\sqrt{\omega^2 - K^2 v_F^2}\right) \end{aligned} \quad (9)$$

4. The solving of Eq. (5) is too complicated if there is an anisotropy in $(\mathbf{k}, \mathbf{n})$ plane. However, only long surface waves (with $k << \Omega/v_F$, k_F) are reachable to direct experimental observation; so it is reasonable to limit consideration to the main approximation in k. At small k the integrand in (5) has a pole at small $|Q|$ ($q_0 \sim ik$; see (4)); the residue in it gives us the term not depending on k , and the rest of integral has to be calculated at zero k . So the dispersion equation is linear in k and can be rewritten in the form

$$k \int_{-\infty+i0}^{+\infty+i0} \frac{dq}{\pi q^2 \epsilon_n(q,\omega)} = -1 - \epsilon_n^{-1}(0,\omega)\sqrt{\frac{\epsilon_k(0,\omega)}{\epsilon_n(0,\omega)}} \quad (10)$$

where ϵ_k and ϵ_n can be ϵ_z or $\epsilon_\perp$ from Eqs. (9). When $\mathbf{k}$ or $\mathbf{n}$ is orthogonal to the layers, the right hand becomes zero at

$$\omega = \Omega_\mu \equiv \Omega\sqrt{\mu/(1+\mu)}, \quad (11)$$

and in the symmetric case $\mathbf{k}, \mathbf{n} \perp 0Z$ we have to set $\mu = 1$: here the spectrum begins from $\Omega/\sqrt{2}$, as in an isotropic metal [1-3]. Note that integral in (10) has imaginary part due to the branch points of functions (9). This describes the Landau collisionless damping, that inevitably occurs in a system whose particles can move in phase with the wave. Finally,

$$k(\omega) \cong \frac{\omega - \Omega_\mu}{V}(1 + i\Gamma) \quad (12)$$

where V is the group velocity and Γ is relative damping of surface wave. Calculating the integral in three principal geometric orientations, we obtain:

$$\mathbf{k} \parallel 0Z : V \approx (\mu/\sqrt{2})v_{z0}; \Gamma \approx -0.449\sqrt{\mu}\ln\sqrt{\mu} \quad (13)$$

$$\mathbf{n} \parallel 0Z : V \approx (\sqrt{3}/2)v_F; \Gamma \approx 0.096\mu^{3/2} \quad (14)$$

$$\mathbf{k}, \mathbf{n} \perp 0Z : V \approx 0.439 v_F; \Gamma \approx 0.032 \quad (15)$$

The velocity is anomalously low for the wave propagating across the layers along a crystal boundary perpendicular to them; such waves could find a technical use for so-called delay lines. We also note the specific smallness (stemming from the smallness of the anisotropy parameter μ) of relative damping Γ in first two cases; but in third case which is qualitatively equivalent to an isotropic metal, the collisionless damping happens to be small only numerically.

5. At last, in retardation region, i.e. at very small k ($k \sim \omega/c$), we obviously can operate in macroscopic way, using the local values of partial dielectric functions (9) in Maxwell equations and matching on the border $E_k^{ext} = E_k^{in}$ and $E_n^{ext} = \epsilon_n(0) E_n^{in}$. In main geometries indicated in (13-15) it gives us the following dispersion relations

$$\begin{aligned} k(\omega) &\cong \frac{\omega}{c}\sqrt{\mu\frac{\Omega^2 - \omega^2}{\mu\Omega^2 - (1+\mu)\omega^2}} \\ k(\omega) &\cong \frac{\omega}{c}\sqrt{\frac{\mu\Omega^2 - \omega^2}{\mu\Omega^2 - (1+\mu)\omega^2}} \\ k(\omega) &\cong \frac{\omega}{c}\sqrt{\frac{\Omega^2 - \omega^2}{\Omega^2 - 2\omega^2}} \end{aligned} \quad (16)$$

respectively.

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