Coherent phenomena in a normal conductor with a superconducting coating

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Abstract

The thermodynamic properties of a mesoscopic-size simply connected cylindrical normal metal with a superconducting coating are studied. It is accepted that a vector potential field can be varied inside the normal layer. The quasiparticles move ballistically through the normal metal and undergo the Andreev scattering by the off-diagonal potential. We find the spectrum of the Andreev levels and calculate the density of states (DOS) of the system. It is shown that the Andreev levels shift as a trapped flux changes inside the normal conductor. At a certain flux value they coincide with the Fermi level. A resonance spike in the DOC appears in this case. As the flux is increased, the DOC behaves as a stepwise function of the flux. The distance between the steps is equal to the superconducting flux quantum $h_c/2e$.

Key words: Andreev reflection; coherent quantum phenomena; Aharonov-Bohm effect; Multidimensional quasiclassical method; resonance spikes

Quantum interference fenomena in normal cylindrical conductors was studied in [1,2]. This study is devoted to coherent quantum phenomena in a mesoscopic-size cylindrical normal conductor contiguous with a superconductor [3].

Let us deduce an equation for the spectrum of quasiparticles (QP) with the energies $E < \Delta$ which move inside a normal cylindric conductor with a superconducting coating. The normal metal is assumed to be pure. Excitations move in it ballistically and experience the Andreev scattering at the NS boundary. When a certain flux Φ is trapped in the normal cylinder, the angular momentum of the "hole" is $\hbar(m + \tilde{\eta}/2)$, while that of the "particle" is to $\hbar(m - \tilde{\eta}/2)$, $\tilde{\eta} = \eta + [\eta]$.

The multi-dimensional quasi-classical Keller and Rubinow's method [4] can be generalized readily for the motion of QP experiencing the Andreev scattering at the circular region boundaries. The wave function in the pre-assigned field of the vector potential $\mathbf{A}$ ($A_r = A_z = 0$; $A_\theta = \Phi/2\pi r$) is a single-valued function $\mathbf{r}$ on a complete circuit over the cylinder surface. With the gradient transformation $\mathbf{A}' = \mathbf{A} + \nabla\chi$ using the function $\chi = +\frac{\hbar c}{2e}[\eta]\theta$, the pairing potential Δ becomes real and the new wave function differs from the former in the factor $(-1)^{[\eta]}$, where $[\eta]$ is the number of quanta of the trapped flux. The phase increment in the wave function of the "particle" on the circular contour of the caustic radius a_0 is $\oint(k_0 + \frac{e}{\hbar c}\mathbf{A}')d\mathbf{s} = 2\pi m$. The phase increment of the wave function of a "hole" can be found similarly on the contour of the radius a_1. As a result, we arrive at the expressions:

$$k_0 a_0 = m - \tilde{\eta}/2, \quad k_1 a_1 = m + \tilde{\eta}/2. \qquad (1)$$

The derivation of the second equation in Eq.(1) takes into account the fact that the effective charge of the "hole" is opposite in sign to the charge of the "particle".

Equation for the spectrum of QP is:

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$$\sqrt{k_0^2R^2-m_0^2}-\sqrt{k_1^2R^2-m_1^2}-m_0\arccos\frac{m_0}{k_0R}+$$

$$+m_1\arccos\frac{m_1}{k_1R}=\pi\left(n+\frac{1}{2}+\frac{1}{\pi}\arccos E/\Delta\right). \quad (2)$$

Here k_0, k_1 are the wave vectors of a "particle" and a "hole", respectively, in the N-layer; $\hbar m_0=\hbar(m-\tilde\eta/2)$, $\hbar m_1=\hbar(m+\tilde\eta/2)$ are their angular momentum, m is absolute value of the magnetic quantum number, $n=0,1,2,\ldots$.

Let us calculate the density of states $\nu(E)$ for the excitations localized in a normal cylinder. In a quasi-classical approximation an analytical solution could be obtained. Expanding the left-hand side of the Eq.(2) in $\frac{1}{k_FR}\ll 1$ we obtain

$$E_n(q)=\frac{\pi\hbar v_r(q)}{2R|\sin\alpha/2|}\left(n-\frac{\tilde\eta}{2\pi}\alpha+\frac{1}{2}+\frac{1}{\pi}\arccos\frac{E}{\Delta}\right). \quad (3)$$

The spectrum Eq.(3) is similar to that of the $S-N-S$ contact obtained by Kulik [5]. This spectrum corresponds to motion of QP between two points on the cylinder surface parametrized by the radial angle α, while the distance between these points is $2R|\sin\alpha/2|$. However, contrary to the standard $S-N-S$ contact case the phase difference $-\frac{\tilde\eta}{2\pi}\alpha$ depends on the flux. For given α, the spectrum Eq.(3) describes a "$S-N-S$" contact with an effective density of states $\nu(E;\alpha)$. The total density of states is given by: $\nu(E)\sim\int_0^{2\pi}\nu(E;\alpha)d\alpha$. The spectrum of Eq. (3) was obtained assuming that the QP energy E was close to the Fermi energy ζ. The Andreev levels of the mesoscopic system can shift when the flux changes; at a certain value of the flux they can coincide with ζ. Within our approximation (3), we should take into account that inside the cylinder cross-section there exist a great number of "$S-N-S$ contacts" with the pre-assigned chord length, i.e., these states possess high degenerate multiplicity about $N\sim 2\pi R/\lambda_B\gg 1$ (λ_B is the de-Broglie wavelength).

The resonance contribution to $\nu(E)$ can be calculated using the spectrum of lower-energy levels

$$\nu(\epsilon)\sim A\epsilon\int_0^{\pi}d\alpha\times$$

$$\times\sum_{n=-\infty}^{+\infty}\frac{\sin^2\frac{\alpha}{2}\theta\left(n+1-\frac{\tilde\eta}{2\pi}\alpha-\epsilon\sin\frac{\alpha}{2}\right)}{\left(n+1-\frac{\tilde\eta}{2\pi}\alpha\right)\sqrt{\left(n+1-\frac{\tilde\eta}{2\pi}\alpha\right)^2-\epsilon^2\sin^2\frac{\alpha}{2}}}, \quad (4)$$

where $A=\frac{8LRm^*}{\pi^2\hbar^2}\frac{2\pi R}{\lambda_B}$, L is the cylinder height, $\theta(x)$ is the step function equal to unity at $x>0$ and to zero at $x<0$. We introduce the dimensionless energy $\epsilon=E/E_0$, $E_0=\frac{\hbar^2}{2m^*R^2}$. As Eq. (4) shows, for a pre-assigned flux, the denominator of the radicand can become zero at a certain angle $\alpha=\alpha(\Phi)$. When $\epsilon\to 0$ $\alpha_n\simeq\frac{(n+1)2\pi}{\tilde\eta}$. In this case the inequalities $0\lesssim\alpha_n\lesssim\frac{(n+1)2\pi}{\tilde\eta}\lesssim\pi$; ($\eta=\Phi/\Phi_0$) are fulfilled, which leads to the condition $\eta+[\eta]\gtrsim 2(n+1)$. Thus, with the pre-assigned flux Φ, the number of terms in the series of Eq. (4) is limited and it increases with growing Φ.

The main contribution to the resonance of the density of states appears near the angle $\alpha=\pi$. Introducing the notions $\xi=\pi-\alpha\ll 1$, $a=n+1-\tilde\eta/2$, $b=\tilde\eta/2\pi$, we obtain the equation for the resonance condition:

$$a^2+b^2\xi^2+2ab\xi-\epsilon^2\left(1-\frac{\xi^2}{4}\right)=0, \quad (5)$$

whose solution is $\xi\simeq -a/b\pm\epsilon$. It is seen that for $b\sim 1$, the energy ϵ and the value a are both small, but the condition $a\gtrsim\epsilon$ is always fulfilled. The expression in brackets before the radical in Eq. (4) is therefore of the order ϵ and is cancelled together with the energy before the integral. The remaining integral is estimated to be a constant of about unity.

The resonance-induced spike of the density of states appears always when the Andreev level coincides with the Fermi energy at a certain flux in the N-layer. The sharp increase in the amplitude $\nu(E)$ at $E\to 0$ is caused by the integral contribution of the states of quasi-particles indexed by the quantum number q, which describe the quasi-particle motion along the sample axis.

When the resonance is disturbed, the condition $(n+1-\frac{\tilde\eta}{2\pi}\alpha)\neq 0$ is fulfilled and for low energies $\epsilon\to 0$ we find $\nu^{(0)}(\epsilon)\sim\epsilon$. Near the resonance, the ratio of the resonance and non-resonance amplitudes of the density of states is: $\nu^{\rm res}/\nu^{(0)}\sim 1/\epsilon\gg 1$. It is thus shown that on variation of the trapped magnetic flux, the density of states of a normal cylindric conductor coated with a thin superconducting layer ($\sim\xi_0$) is described by the step function Φ. The step spacing is equal to a superconducting flux quantum Φ_0 and the step height increases with the flux.

To conclude, we note that the mean free path of the quasi-particles was expected to be the largest parameter of the problem. The allowance for the scattering by impurities will decrease the amplitude of the resonance spikes.

References

[1] I.O. Kulik. Pis'ma Zh. Eksp. Teor. Fiz. **11**, 407 (1970) [JETP Lett. **11**, 275 (1970)]

[2] E.N. Bogachek and G.A. Gogadze. Zh. Eksp. Teor. Fiz. **63**, 1839 (1972) [Sov. Phys. JETP **36**, 973 (1973)]

[3] G.A.Gogadze, R.I.Shekhter and M.Jonson. Low Temp. Phys. **27**, 912 (2001)

[4] J.B. Keller and S.I. Rubinow. Ann. Physics **9**, 24 (1960)

[5] I.O. Kulik. Zh. Eksp. Teor. Fiz. **57**, 1745 (1969) [Sov. Phys. JETP **30**, 944 (1969)]