BCS-BEC Crossover in a Trapped Gas of Fermi Atoms with a Feshbach Resonance

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Abstract

We present a theoretical study of the BCS-BEC (Bose-Einstein condensation) crossover in a trapped gas of Fermi atoms including the effect of a Feshbach resonance. Going past the simple mean field approximation, we include the effect of fluctuations in the strong coupling limit involving two kinds of Bose molecules, i.e., Feshbach molecules associated with the resonance and preformed Cooper-pairs in which the pairing interaction is mediated by Feshbach quasi-molecules. The superfluid phase transition is shown to change continuously from the BCS-type to a BEC of these two kinds of Bosons, as the threshold energy of the Feshbach resonance is lowered.

Key words: BCS-BEC crossover; Feshbach resonance; cold gas of Fermi atoms; superfluidity

Superfluidity in ultracold gases of Fermi atoms is one of the hottest topics in current physics research. At present, Fermi gases (^{40}K and ^{6}Li) have been cooled to $T \sim 0.2T_{\rm F}$ (where $T_{\rm F}$ is the Fermi temperature).[1] Recent work[2,3] pointed out that a Feshbach resonance can enhance the pairing mechanism for Cooper-pairs in such atomic systems.

The Feshbach resonance originates from the hyperfine interaction. In this resonance, a quasi-molecule is formed and this can give rise to an attractive interaction between the Fermi atoms. Since this interaction becomes very strong when the threshold energy of the resonance is tuned optimally, we can expect superfluidity to occur at a higher transition temperature $T_{\rm c}$. However, such a strong pairing interaction is also known to enhance superfluid fluctuations, which in turn suppress $T_{\rm c}$ predicted by a simple weak-coupling BCS theory.[4] Thus in considering "high-$T_{\rm c}$" superfluidity induced by a Feshbach resonance, we have to include these fluctuation contributions. In this paper, we discuss such strong fluctuation effects on $T_{\rm c}$ in a gas of Fermi atoms with a Feshbach resonance.[5,6]

This system can be described by the coupled Fermion-Boson model Hamiltonian[7,2,3]

$$H = \sum_{\mathbf{p}\sigma} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_{\mathbf{q}} (E_{\mathbf{q}} + 2\nu) b^\dagger_{\mathbf{q}} b_{\mathbf{q}} - U \sum_{\mathbf{p},\mathbf{p}'} c^\dagger_{\mathbf{p}\uparrow} c^\dagger_{-\mathbf{p}\downarrow} c_{-\mathbf{p}'\uparrow} c_{\mathbf{p}'\downarrow} + g \sum_{\mathbf{p},\mathbf{q}} [b^\dagger_{\mathbf{q}} c_{-\mathbf{p}+\mathbf{q}/2\downarrow} c_{\mathbf{p}+\mathbf{q}/2\uparrow} + {\rm h.c}] + V_{\rm trap}. \quad (1)$$

Here $c^\dagger_{\mathbf{p}\sigma}$ is the creation operator of a Fermi atom with the kinetic energy $\varepsilon_{\mathbf{p}} = p^2/2m$. The pseudo-spin variable $\sigma = \uparrow, \downarrow$ describes two hyperfine states responsible for superfluidity. Creation of the (Feshbach) molecular Boson by the resonance is expressed by $b^\dagger_{\mathbf{q}}$ and $E_{\mathbf{q}} + 2\nu = q^2/2M + 2\nu$ is the kinetic energy of this Boson. The bottom of this Bose energy spectrum, 2ν, is also referred to as the threshold energy. Since one molecular b-Boson consists of two Fermi atoms with the mass m, we take $M = 2m$. This condition also leads to the conservation of the total number of Fermi atoms; this

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constraint can be incorporated into the model Hamiltonian (1) by replacing $\varepsilon_{\mathbf{p}} \to \varepsilon_{\mathbf{p}} - \mu$ and $E_{\mathbf{q}} \to E_{\mathbf{q}} - 2\mu$,[5] where μ is the chemical potential. The interaction term in (1) including g describes the Feshbach resonance, in which two Fermions form a b-Boson and the related decay of a b-Boson into two Fermions. The Hamiltonian also includes the attractive interaction $-U < 0$ originating from non-resonant processes. The last term $V_{\rm trap} \equiv \sum_j \frac{1}{2} m\omega_0^2 r_j^2$ is an isotropic harmonic potential, where r_j is the position of the j-th atom.

We take into account the effect of superfluid fluctuations within the Gaussian approximation, extending the theory developed by Noziéres and Schmitt-Rink.[4] In the present case, we include infinite order processes involving the Feshbach coupling interaction g, as well as the usual superfluid Cooper-pair fluctuations caused by U.[5] As for the effect of $V_{\rm trap}$, we employ the usual local density approximation (LDA). Within this LDA, $T_{\rm c}$ is determined from the Thouless criterion by the condition that the phase transition first occurs at the center of the trap potential (largest density). The chemical potential μ is determined from the thermodynamic potential including the effect of superfluid fluctuations within the Gaussian approximation. We solve these two equations for $T_{\rm c}$ and μ self-consistently.[5,6]

The BCS-BEC crossover behavior is shown in Fig.1(a). As the threshold energy 2ν decreases, $T_{\rm c}$ continuously changes from the BCS-type to the BEC-type. In the BEC regime, $T_{\rm c}$ is much smaller than the value expected from the weak-coupling BCS theory because of the effect of the fluctuations; as expected, $T_{\rm c}$ approaches $0.518\varepsilon_{\rm F}$ in the formal limit $\nu \ll -\varepsilon_{\rm F}$, where $\varepsilon_{\rm F}$ is the Fermi energy of the free Fermi gas.

As the threshold energy 2ν is lowered (namely, the gas approaches the BEC regime), stable preformed Cooper-pairs ($N_{\rm C}$) appear as shown in Fig. 1(b). This phenomenon is the same as that discussed by Noziéres and Schmitt-Rink in strong coupling superconductivity.[4] However, in the present case, besides this kind of Boson (preformed Cooper-pairs), Feshbach-induced molecules also become stable in the BEC regime ($N_{\rm B}$). In the limit $\nu \ll \varepsilon_{\rm F}$, these Bosons are seen to become the dominant excitations. This situation is in contrast to the case of ordinary superconductivity (where $g = 0$), where the preformed Cooper-pairs are the dominant Bosons in the BEC regime.[4]

The change of the character of the particles from Fermi atoms ($N_{\rm F}^0$) to composite Bosons ($N_{\rm B} + N_{\rm C}$) also affects the spatial distribution of atoms in the trap (see inset in Fig. 1.). Because of the Pauli exclusion principle, the spatial distribution of Fermi atoms is spread out for $\nu = \varepsilon_{\rm F}$ (where Bosons are almost absent). However, as ν decreases, the atoms form Bosons and these are seen to cluster at the center of the trap, due to their Bosonic character.

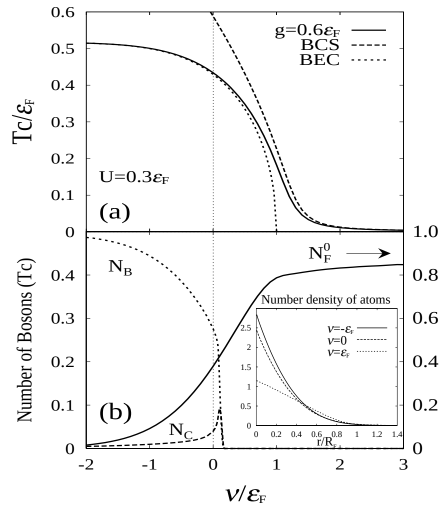

Fig. 1. (a) Dependence of $T_{\rm c}$ on the Feshbach threshold 2ν for Fermions in a harmonic trap.[6] BCS represents the mean field BCS limit[3] and BEC gives $T_{\rm c}$ for $N/2$ non-interacting Bosons in a trap. (b) ν-dependence of the number of free Fermions $N_{\rm F}^0$, stable Feshbach molecules $N_{\rm B}$ and stable preformed Cooper-pairs $N_{\rm C}$. Inset: Density profile at $T_{\rm c}$. $R_{\rm F}$ is the radius of the gas at $T = 0$ in the case of free Fermi atoms. One molecule counts as two atoms. The contributions of the scattering states and the Feshbach resonating quasi-molecules are included.

In conclusion, we have discussed the BCS-BEC crossover in a trapped gas of Fermi atoms with a Feshbach resonance. In the BEC regime, two kinds of stable Bosons appear. This crossover phenomenon affects the spatial distribution of atoms, which may be useful in detecting the formation of these Bosons experimentally. The collective mode spectrum below $T_{\rm c}$ will be discussed elsewhere.[6]

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References

[1] B. DeMarco and D. S. Jin, Science **285** (1999) 1703; A. G. Truscott et al., Science **291** (2001) 2570; F. Schreck et al., Phys. Rev. Lett. **87** (2001) 080403.

[2] E. Timmermans et al., Phys. Lett. A **285** (2001) 120406; Phys. Rep. **315** (1999) 199.

[3] M. Holland et al., Phys. Rev. Lett. **87** (2001) 120406; M. L. Chiofalo et al., Phys. Rev. Lett. **88** (2002) 090402.

[4] P. Noziéres, S. Schmitt-Rink, J. Low. Temp. Phys. **59**, 195 (1985). For a review, see M. Randeria, in *Bose-Einstein Condensation*, ed. A. Griffin et al. (Cambridge, N.Y. 1995) p.355.

[5] Y. Ohashi, A. Griffin, cond-mat/0201262.

[6] Y. Ohashi, A. Griffin, in preparation.

[7] J. Ranninger, in *Bose-Einstein Condensation*, ed. A. Griffin et al. (Cambridge, N.Y. 1995) p.393; R. Friedberg, T. D. Lee, Phys. Rev. B. **40**, 6745 (1989).