Single parameter scaling of the conductance distribution in mesoscopic conductors

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Abstract

The distribution of conductance g in phase coherent mesoscopic conductors is investigated near the Anderson transition. The distribution function $P(g)$ in 3D orthogonal systems shows single parameter scaling, which reconciles the phenomenon of the universal conductance fluctuation with the scaling theory of localisation.

Key words: Disordered systems ; Anderson localisation ; Scaling Theory

1. Introduction

The most revolutionary idea in the theory of disordered systems, the scaling theory of localisation [1], is now over twenty years old. It has been known for almost as long that the central hypothesis of the scaling theory, that the conductance of disordered system obeys a one parameter scaling law, is not correct. It was realised shortly after the proposal of the scaling theory that the scaling hypothesis is inconsistent with phenomenon of mesoscopic conductance fluctuations. Instead, it was generally thought that the scaling hypothesis must apply to some average of the conductance distribution of perhaps to the distribution of conductance itself. Yet, until our recent paper [2], no demonstration that this is the case, or an identification of which average or averages obey the scaling hypothesis, has ever been presented. Indeed, in the classic numerical work on Anderson localisation [3], in which the localisation of electrons on quasi-one dimensional system was analysed, the conductance was never calculated and only an indirect verification of scaling hypothesis was possible.

In our recent paper [2] we simulated ensembles of three dimensional disordered systems and studied their zero temperature conductance distribution. We presented an explicit demonstration that both the mean resistance, the mean conductance and the mean of the logarithm of the conductance all obey single parameter scaling laws. We were further able to show that, even though the β-function for each average is different, the critical exponents deduced from the scaling of each average are the same.

In this contribution we analyse the scaling of the conductance distribution rather than its averages. Naively it might be thought that scaling of the averages necessarily implies scaling of the distribution. Unfortunately there is no logical justification for this statement. For example, a Guassian distribution may have a mean and variance which obey independent scaling laws or the variance might not even obey a scaling law. Thus, while our results for the scaling of different averages of the conductance distribution are suggestive, they are not conclusive as far as the claim that the distribution of conductance obeys a one parameter scaling law is concerned. Here we eliminate such possibilities and demonstrate conclusively that the conductance distribution of a disordered systems obeys a one parameter scaling law.

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2. Method

A one parameter scaling law for the conductance distribution $p(g)$ of a three dimensional system of linear dimension L can be formulated as follows:

$$p(g) \simeq F(g;X), \qquad \frac{\mathrm{d}\ln X}{\mathrm{d}\ln L} = \beta(X). \tag{1}$$

The parameter X need not be one of the moments of the distribution.

Attempting to verify (1) directly is not the best approach. Instead, we analyse the scaling of the percentiles of the conductance distribution. The precise definition of the percentile g_q is

$$q = \int_0^{g_q} p(g)\mathrm{d}g \tag{2}$$

where $q \in [0,1]$. A demonstration that all the percentiles obey consistent one parameter scaling laws is equivalent to a demonstration of (1).

As a model Hamiltonian we take

$$H = V \sum_{<i,j>} C_i^\dagger C_j + \sum_i W_i C_i^\dagger C_i, \tag{3}$$

where $C_i^\dagger$ (C_i) is the creation (annihilation) operator of an electron at the site i of a three dimensional cubic lattice. The amplitude of the random potential at site i is W_i. Hopping is restricted to nearest neighbours and its amplitude is taken as the unit of energy, $V = 1$. We assume a box distribution with each W_i uniformly distributed on the interval $[-W/2, W/2]$. We refer to the strength of the potential fluctuations W as the disorder. We evaluate the zero temperature two terminal conductance $g_\mathrm{L} = 2\mathrm{tr}t^\dagger t$, where t is the transmission matrix describing the propagation of electrons between opposite faces of the system, using the method of [4]. We subtract the contact resistance from g_L to obtain g.

3. Results

In this short paper we present results for the median conductance only ($q = 1/2$); our results for other percentiles will be presented in a longer paper. In an analogous manner to that described in [2] we fit the disorder and size dependence of the median conductance to a one parameter scaling law. The Fermi energy is set at $E_F = 0.5V$. The results are displayed in Figure 1 and 2. The lines in the figures are a 10 parameter fit to 211 data points. The χ^2 statistics is 201 corresponding to a goodness of fit probability of 0.4. The critical disorder at which the metal-insulator transition occurs is estimated to be $W_c = 16.48 \pm .01$. The estimate of the critical exponent describing the divergence of the localisation length is $\nu = 1.58 \pm .01$. The critical value of the median conductance is $0.340 \pm .04$ in units of e^2/h. The confidence intervals given are at the 95% level. The estimate of the exponent is consistent with that obtained from the scaling of the average conductance [2].

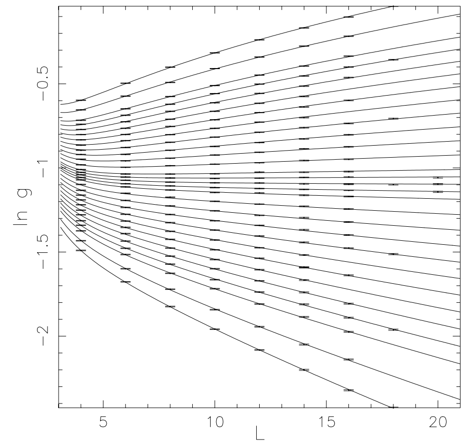

Fig. 1. Size dependence of the median conductance for disorder $W \in [15, 18]$

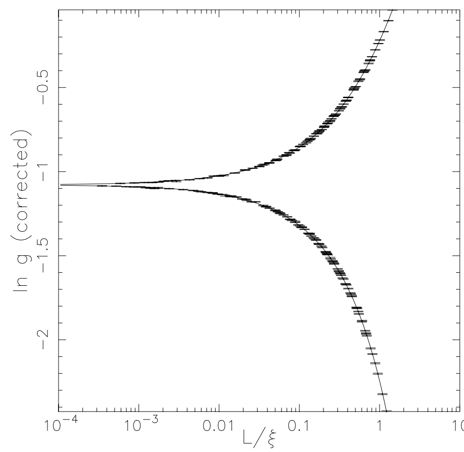

Fig. 2. The same data as in Fig. 1 after removal of corrections to scaling, re-plotted to demonstrates one parameter scaling.

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