A Realistic Model of Spin-Transport in dilute ^{3}He in ^{4}He in a Finite Geometry

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Abstract

We have investigated the spin-motion of spin-polarised solutions of Fermi-liquid ^{3}He in ^{4}He below 100mK. The linearised version of the equation of motion is solved for a finite cylindrical geometry with fully spin-reflecting boundary conditions, and is used to model an actual NMR spin-echo experiment. We include non-linear gradients as well as the applied, linear gradient and we have also incorporated the effect of finite-amplitude NMR excitation pulses. We find that the presence of boundaries causes marked deviation from the analytical expressions previously used for data analysis. The non-uniform gradient causes spin-echoes to be delayed and it is noted that the finite amplitude pulses also have a significant effect on the appearance and timing of spin-echoes. Previous values for the 'anisotropy temperature' in the spin-polarised Fermi-liquid, derived experimentally from spin-echoes, are shown to be overestimates.

Key words: 3He-4He mixtures, spin-polarised helium, NMR, spin-waves

1. Introduction

Two types of experiment have been used to determine whether or not the transverse-relaxation time in spin-polarized ^{3}He systems saturates at low temperature. The saturation is characterised by an 'anisotropy temperature', T_a, which is expected to be magnetic field dependent. Spin echoes have yielded values for T_a in the range 10 and 25mK[1] when normalised to 12 Tesla, in contrast to spin-wave experiments, which show no saturation[2].

Here, we show that the analysis used for spin-echo experiments in mixtures must take account of restricted diffusion. A numerical simulation is developed, including a non-linear field variation and NMR pulses of finite duration and amplitude. For pure ^{3}He experiments the boundaries are less important, but the demagnetising field cannot be neglected.

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2. Restricted Diffusion

The equation of motion, for components of magnetisation transverse to the applied magnetic field, is linear for small tipping-angles[4].

$$\frac{\partial m_-}{\partial t} = \mathrm{i}\gamma B(\mathbf{r}) m_- + \frac{D_\perp}{(1+\mathrm{i}\mu M_0)} \nabla^2 m_- \qquad (1)$$

Here, $m_- = m_x + \mathrm{i}m_y$ is a combination of the transverse components of magnetisation; $\gamma B(\mathbf{r})$ is the local Larmor frequency relative to the Larmor frequency at the centre of coordinates; $D_{\mathrm{eff}} = \frac{D_\perp}{(1+\mathrm{i}\mu M_0)}$ is the effective diffusion coefficient, including the effect of spin-rotation. Note that the magnetic field is assumed to have components along the z-axis only.

The boundary condition, at the surface of the experimental cell, is fully reflecting for spin-transport, i.e. $(\hat{\mathbf{n}}.\nabla)m_- = 0$, with $\hat{\mathbf{n}}$ being the unit vector perpendicular to the wall. We have solved this equation of motion in a cylinder, numerically.

Leggett and Rice[4] showed how spin-echo experiments can be analysed to obtain the spin-rotation pa-

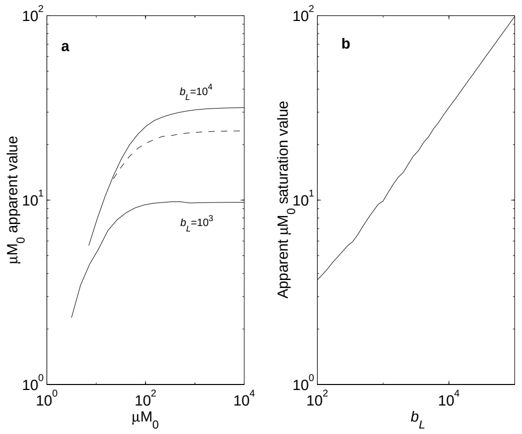

Fig. 1. (a)Slope of spin-echo phase as function of logarithm of amplitude, versus μM_0 in simulation, for three different dimensionless field variations.(b)The saturation value of μM_0 varies with field gradient. A typical experiment has $b_L = \gamma G L^3 \mu M_0 / D_\perp \simeq 10000$, where L is half the length of the cell, and G is the linear field gradient.

rameter, μM_0. A graph is plotted according to the formula

$$\phi = -\mu M_0 ln|h| \tag{2}$$

where ϕ and h are respectively the phase and amplitude of the echo. The slope of this graph gives us a measure of μM_0. This analysis assumes an unbounded sample volume with a purely linear field gradient. Our simulation shows that this leads to underestimation of μM_0 at low temperatures (large μM_0), as seen in figure 1(a), which shows the slope against the spin-rotation parameter used in the simulation. The limiting graph slope is shown as a function of the gradient parameter, b_L in figure 1(b), so the bigger the gradient, the stronger the effect. This effect mimics low-temperature saturation of the transverse-spin relaxation time.

Furthermore, non-uniform gradients change not only the amplitude and phase of spin-echoes, but also their timing and shape, an effect which is seen in simulation. In 6.2% ^{3}He in ^{4}He we find experimentally that the echoes are delayed, agreeing with the prediction of the simulation for this sign of μM_0 (negative).

3. Finite Duration π-pulses

A realistic simulation must include the effect of NMR pulses of finite duration and amplitude. These are modelled as rotations about the local field direction[6]. This contrasts with the assumption commonly used - that a tipping pulse is results in a uniform m_-, and a perfect π-pulse causes reversal of the sign of μM_0 and complex-conjugation of m_-. When π-pulses are much shorter

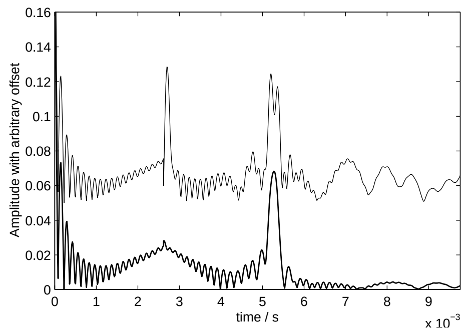

Fig. 2. Simulations of spin-echo experiment. The timescale in the simulation is $\frac{1}{\gamma L G} = 43\mu s$. The pulse durations for the lower and upper curves are 1 and 30μs respectively, the latter of which is normal for a real experiment. Note the distortion of the echo.

than the inverse of the frequency variation across the cell, the assumption is accurate, but this is not the case for real experiments (see figure 2).

4. Conclusions

To analyse data from real experiments on spin-transport correctly, one needs a detailed model of the spin-dynamics. There are several important effects - e.g. restricted diffusion, non-uniform gradients, finite NMR pulse durations - which make analytical formulae invalid. We have used this model to fit experimental data to find the real, bulk transport coefficients, to detect any real saturation of $D_\perp$ or μM_0 at low temperature[3]. We conclude that the anisotropy can be characterised with a temperature $T_a = 6$mK, in an 12T field.

References

[1] L-J Wei, N. Kalechofsky, D. Candela, *Phys. Rev. Lett.* **71**, 879 (1993); J.H. Ager, A. Child, R. Konig, J.R. Owers-Bradley, R.M. Bowley, *J. Low Temp. Phys.* **99**, 683 (1995); J.R. Owers-Bradley, R.M. Bowley, O. Buu, D. Clubb, G. Vermeulen, *J. Low Temp. Phys.* **121**, 779 (2000); D. Candela, H. Akimoto, R.M. Bowley, O. Buu, D. Clubb, J.R. Owers-Bradley, *J. Low Temp. Phys.* **121**, 767 (2000); H. Akimoto, J.S. Xia, E.D. Adams, D. Candela, W.J. Mullin, N.S. Sullivan, *J. Low Temp. Phys.* **126**, 109 (2002).

[2] G.A. Vermeulen, A. Roni, *Phys. Rev. Lett.* **86**, 248 (2001).

[3] O.Buu, D. Clubb, R. Nyman, R.M. Bowley, J.R. Owers-Bradley, to appear in *JLTP*.

[4] A.J. Leggett, M.J. Rice, *Phys. Rev. Lett.* **20**, 586 (1968); A.J. Leggett, *J. Phys. C*, **3**, 448 (1970).

[5] O.Buu, R. Nyman, R.M. Bowley, J.R. Owers-Bradley, *Phys. Rev. B.* **65**, 134512 (2002).

[6] E.T. Jaynes, *Phys. Rev.* **98**, 1099 (1955); A.L Bloom, *Phys. Rev.* **98**, 1105 (1955).