Volume flow in liquid ^{3}He in the Knudsen and Poiseuille regions

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Abstract

We have measured the volume flow rate in response to a pressure gradient of normal liquid ^{3}He through 210 nm diameter cylinders, in both the Knudsen and Poiseuille regions. Because of the regular geometry, the data are directly comparable with theoretical calculations, and we find good agreement with theory. In particular, the temperature and depth of the Knudsen minimum agree well with theory. We increased the specular scattering coefficient by adding monolayers of ^{4}He; the flow rate results agreed with what was expected from previous measurements.

Key words: helium-3; normal; confined; transport

Normal liquid ^{3}He is a good system to study the effects of the bulk mean free path, λ, on transport coefficients because λ may be varied by a factor 10^4 by changing the temperature T. We present measurements of the quasiparticle flow rate through cylinders of diameter $d = 210$ nm; changing T from 1 to 100 mK changes λ from 50 μm to 5 nm and we thus covered the Knudsen ($\lambda \gg d$), slip ($\lambda \sim d$) and Poiseuille ($\lambda \ll d$) regions (at saturated vapour pressure). Furthermore, by pre-plating the cylinders with 2 and 4 nominal monolayers (ML) of ^{4}He, we were able to vary the specular scattering coefficient ν at the boundaries. These are the first such measurements in ^{3}He in the Knudsen region with a regular geometry, enabling direct comparison with theory; previous experiments with ^{3}He in the Knudsen regime used the irregular geometry of a packed powder [1].

The flow rates for a Maxwell gas have long been known [2]; the theoretical results for a normal Fermi liquid are analogous [3]. The volume flow rate $\dot{V}$ is proportional to the pressure difference ΔP; in the Poiseuille region for one cylinder of length l, $\dot{V}/\Delta P = \pi d^4/128\eta l$, where $\eta \propto \lambda$ is the viscosity. In ^{3}He, $\eta \propto 1/T^2$ and $\dot{V}/\Delta P \propto T^2$. As λ is increased, the effects of slip at the boundary are taken into account by increasing the effective diameter to $d + 2\zeta$, where ζ is the slip length. For ^{3}He the theoretical result is $\zeta = 0.582\lambda$ [4]. In the Knudsen region $\dot{V}/\Delta P = \pi d^3/4m^* v_F n l$, where m^*, v_F and n are the ^{3}He effective mass, Fermi velocity and number density respectively; $\dot{V}/\Delta P$ is therefore expected to be independent of temperature. When $\lambda \sim d$, $\dot{V}/\Delta P$ reaches the Knudsen minimum, as observed in ^{3}He with parallel plates [5]. Changing the boundary conditions such that a fraction ν of quasiparticles are specularly scattered ($\perp$ momentum changes sign, $\parallel$ momentum unchanged) increases $\dot{V}$ in the Knudsen region and ζ by a factor $(1 + \nu)/(1 - \nu)$.

Our experimental cell [6] was 2 chambers of ^{3}He separated by a stack of 11 Anopore filter membranes, spaced with 50 μm thick Mylar washers. SEM photographs showed that the cylinders in the membranes were circular, uniform and non-intersecting, with no sign of any barrel shape. Each disk was 60 μm thick and the area was reduced to give $N=10^8$ cylinders. To create and measure the pressure difference across the cylinders, we used a parallel plate capacitor, one plate of which was a 20 mm diameter gold-plated Kapton membrane separating the chambers. The volume and pressure changes of the chambers were proportional to the capacitance change; the sensitivity was $\Delta C/\Delta P = 4.75\times10^{-15}$ F/Pa. Pressure differences up to 2 Pa could be generated by applying or removing a DC voltage,

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which led to an exponential (over 3 orders of magnitude, the entire measured range) decay of capacitance with time constant $\tau \propto \Delta P/\dot{V}$.

In Fig. 1 we plot τ vs. T for the three different ML coverages. Consider first the results with pure ^{3}He, shown as diamonds. Four regions are distinguishable: (i) Between 0.6 and 6 mK the liquid was in the Knudsen regime and τ was constant. The only two unknowns in the calculation of τ were N and ν; assuming ν=0 allowed us to determine N precisely. (The spike at the bulk superfluid transition temperature 0.93 mK is an artefact resulting from a superleak in 4 out of the 11 disks. Because τ was small for the superfluid we scaled τ below 0.93 mK by 11/7.) (ii) Around 10 mK, τ rose by 5%, corresponding to the Knudsen minimum at 11±2 mK. Curve (g) is the theoretical flow rate [7] in the slip region. The depth and temperature of the measured minimum, corresponding to λ=(3.5±0.5)d, agree well with theory (λ=3.3d), in contrast to the results with parallel plates [5] where λ at the minimum was 1.5× the theoretical value. (iii) Above 20 mK $\lambda \gg d$ and the liquid was in the Poiseuille region where $\tau \propto 1/T^2$. Curve (d) shows the expected result for Poiseuille flow with slip and ν=0. The measured τ were less than the theory; using ν=0.2 or increasing the slip length by 50% from its theoretical value gave good agreement. (iv) Below 0.6 mK τ fell as the liquid entered the superfluid phase. The suppression of T_C was consistent with theory [8].

Next consider the results with 2 ML. τ was qualititavely similar to that with 0 ML. In the Knudsen regime τ was reduced from the 0 ML results by a factor of 2, corresponding to an increase in ν from 0 to 0.33. The Knudsen minimum was much more pronounced with 2 ML, with a flow rate reduction of 30% from the Knudsen region. The temperature of the minimum increased to 25 mK. The calculation [7] does not admit a specularity coefficient so the data could not be compared with theory. At higher temperatures, Poiseuille flow with slip and ν=0.33 describe the data well; in contrast with the 0 ML results the same ν was found in both the Knudsen and Poiseuille regions. The superfluid transition in the cylinders was increased to 0.6 mK and sharpened relative to 0 ML, consistent with theory [8]. Below 0.6 mK $\tau < 1$ s indicating that all the liquid in the cylinders was superfluid. With 4 ML, τ was only weakly temperature dependent and there was no Knudsen minimum. The liquid therefore did not enter the Knudsen limit, but the flow rate was not described by the Poiseuille equation (curve (f)), either: slip theory is a first order correction to hydrodynamics, but with 4 ML $\zeta \gg d$. The magnitude of τ gave ν=0.9. The values of ν for the three coverages are consistent with those determined with a parallel plate geometry [9], where ν=0.2, 0.6, and 0.9 for 0, 2, and 4 ML respectively; we assumed ν=0 for 0 ML and determined

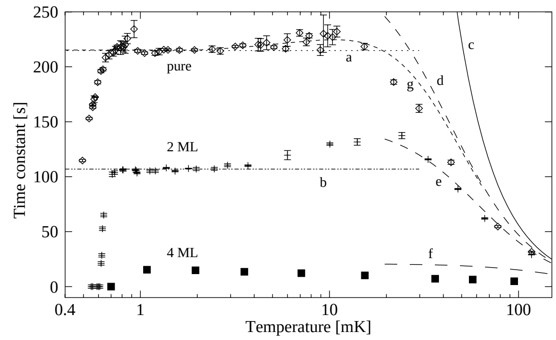

Fig. 1. Time constant $\tau \propto \Delta P/\dot{V}$ for mechanical flow of ^{3}He through the Anopore cylinders. (◇): pure ^{3}He. (+): 2 ML ^{4}He. (■): 4 ML ^{4}He. Lines (a) and (b) are the expected Knudsen flow with ν=0 and 0.33. Curves (c)-(f) are Poiseuille flow with no slip; slip, ν=0; slip, ν=0.33; and slip, ν=0.9. Curve (g) is the theoretical τ in the vicinity of the Knudsen minimum [7].

ν=0.33 and 0.9 for 2 and 4 ML.

In conclusion, we have measured the volume flow rate of normal liquid ^{3}He in 210 nm diameter circular cylinders, in the Knudsen, slip and Poiseuille regions. The results are consistent with theory, with small discrepancies. In particular we have observed a temperature-independent flow rate in the Knudsen region where $\lambda \gg d$, the first such measurements in normal liquid ^{3}He with a regular geometry.

Acknowledgements

The support of NSERC and the technical assistance of Kim MacKinder and Gary Contant are gratefully acknowledged.

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