Spin-current induced Hall effect in superconductors

S. Takahashi [1] and S. Maekawa

Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan

Abstract

Anomalous Hall effect induced by a spin-polarized current in superconductors (SC) is theoretically studied. The spin-polarized quasiparticles flowing in SC are deflected by spin-orbit impurity scattering to create the charge accumulation in the transverse direction. Due to overall charge neutrality, a compensating change appears in the pair density of the condensate. To maintain the electrochemical potential of the condensate, an electric field builds up in the transverse direction, yielding the Hall voltage. It is shown that the Hall voltages due to side jump and skew scattering have different temperature dependence in the superconducting state.

Key words: Hall effect; Spin-polarized transport; Spin injection

The basic mechanism underlying the anomalous Hall effect is the spin-orbit interaction in metals, which causes a spin-asymmetry in the scattering of conduction electrons by impurities; up-spin electrons are preferentially scattered in one direction and down-spin electrons in the opposite direction [1]. As a consequence, the spin-polarized electrons flowing in a nonmagnetic metal are deflected to induce an excess charge on the side of the sample, yielding a transverse Hall voltage [2–5]. The spin-polarized current flowing in superconductors (SC) is of particular interest because SC is a coupled system of the condensate (Cooper pairs) and the quasiparticles, where spin and charge imbalance plays a central role in the spin-dependent transport [6,7]. When the spin-polarized current induces the quasiparticle charge imbalance, the imbalance is compensated by the Cooper pair charge to maintain overall charge neutrality, and thus the Cooper pairs participate in the anomalous Hall effect.

We consider a spin-injection Hall device shown in Fig. 1. The left and right electrodes are FMs and their magnetizations are aligned parallel and point to the z direction. The central electrode is SC with thickness d and width w. If the thickness d of SC is smaller than the spin diffusion length λ_s, the spin-polarized current j_S in SC flows uniformly through SC with little spin accumulation, and its magnitude is given by $j_S = P j_{\rm inj}$ [8], where P the tunneling spin polarization and $j_{\rm inj}$ is the injection current density. Using the Boltzmann transport theory and incorporating the anomalous velocity contribution as well as the modification of QP's distribution function due to the spin-orbit scattering by nomagnetic impurities, we obtain the total charge current flowing in SC [5]

$$\mathbf{j}_Q^{tot} = \mathbf{j}_p + \mathbf{j}_Q + \eta_{\rm SJ}\,[\hat{\mathbf{z}} \times \mathbf{j}_S] - \eta_{\rm SS} eD\,[\hat{\mathbf{z}} \times \nabla S]\,. \qquad (1)$$

Here, $\mathbf{j}_p$ is the supercurrent carried by pairs, $\mathbf{j}_Q$ and $\mathbf{j}_S$ are the longitudinal QP currents driven by the gradient in the chemical potential shifts, $\delta\mu_Q$ and $\delta\mu_S$, of the QP charge and spin: $\mathbf{j}_Q = -2(\sigma_N/e) f_\Delta \nabla \delta\mu_Q$ and $\mathbf{j}_S = -2(\sigma_N/e) f_\Delta \nabla \delta\mu_S$ with $f_\Delta = 1/[\exp(\Delta/k_BT)+1]$, S is the QP's spin density: $S = 2N(0)\chi_S \delta\mu_S$ [7] where $\chi_S \approx 1 - [7\zeta(3)/4\pi^2]\,(\Delta/k_BT)^2$ for $\Delta \ll k_BT_c$ near T_c (Δ is the superconducting gap), and D is the normal-state diffusion constant. In Eq. (1), the second and third terms are the Hall currents due to *side jump* (SJ) and *skew scattering* (SS), having the coefficients $\eta_{\rm SJ}$ and $\eta_{\rm SS}$ proportional to the spin-orbit coupling.

In the open circuit condition in the transverse direction, the y component of Eq. (1) vanishes: $[\mathbf{j}_Q^{tot}]_y = 0$.

[1] E-mail:takahasi@imr.tohoku.ac.jp

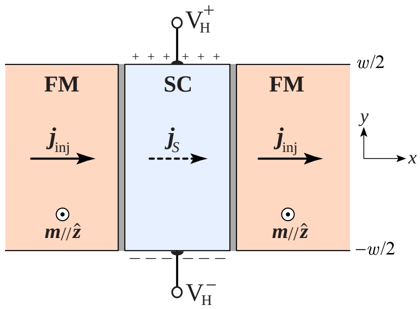

Fig. 1. Schematic diagram of a spin injection device FM/SC/FM which generates the Hall voltage $V_{\rm H}$ in the transverse y direction. The magnetizations of FMs are parallel and point to the z direction, and the spin-polarized current $j_S = Pj_{\rm inj}$ flows through SC.

Using $\partial_y^2 \delta\mu_Q = \delta\mu_Q/\lambda_Q^2$ (λ_Q is the charge diffusion length) and the boundary condition $j_p^y = 0$ at the edge of $y = \pm w/2$, we obtain

$$\delta\mu_Q = \frac{e}{2\sigma_N f_\Delta}\left(\eta_{\rm SJ} + \frac{\chi_S}{2f_\Delta}\eta_{\rm SS}\right)\frac{\sinh(y/\lambda_Q)}{\cosh(w/2\lambda_Q)}j_S.$$

The induced QP's charge $Q_n = 2N(0)\chi_Q\delta\mu_Q$ [9], where $\chi_Q \approx 1 - (\pi\Delta/4k_BT) + [7\zeta(3)/4\pi^2](\Delta/k_BT)^2$ for $\Delta \ll k_BT_c$ near T_c, is compensated by the change in the pair charge Q_p due to charge neutrality in SC. The change of Q_p from its equilibrium value is $\delta Q_p = 2N(0)\delta\mu_p$, where $\delta\mu_p = \mu_p - \epsilon_F$ is the shift in the chemical potential of the pairs from the equilibrium value ϵ_F. Thus the charge neutrality ($Q_n + \delta Q_p = 0$) leads to the relation: $\chi_Q\delta\mu_Q + \delta\mu_p = 0$.

In a stationary state, the electrochemical potential $\Phi_p = \mu_p + e\phi$ for the condensate must be constant throughout SC, where ϕ is the electric potential; Otherwise, the Cooper pairs are accelerated by the force $-\nabla\Phi_p$. Consequently, ϕ is induced in the transverse direction according to $e\phi = \chi_Q\delta\mu_Q$, which yields the Hall voltage $V_{\rm H} = V_{\rm H}^+ - V_{\rm H}^-$ [$V_{\rm H}^\pm = \phi(\pm\frac{w}{2})$]:

$$V_{\rm H} = P\left[\tilde{\eta}_{\rm SJ}(T) + \tilde{\eta}_{\rm SS}(T)\right]G_w w\rho_N j_{\rm inj}, \qquad (2)$$

with $\rho_N = 1/\sigma_N$, $G_w = (2\lambda_Q/w)\tanh(w/2\lambda_Q)$, and

$$\tilde{\eta}_{\rm SJ}(T) = \frac{\chi_Q}{2f_\Delta}\eta_{\rm SJ}, \quad \tilde{\eta}_{\rm SS}(T) = \frac{\chi_S\chi_Q}{(2f_\Delta)^2}\eta_{\rm SS}.$$

In the limit of $T \to T_c$, $\lambda_Q = \lambda_Q^0(1 - T/T_c)^{-1/4}$[10] $\to \infty$, so that Eq. (2) reduces to $V_{\rm H} = P(\eta_{\rm SJ}+\eta_{\rm SS})w\rho_N j_{\rm inj}$ in the normal-state. Recently, spin-dependent Hall effect has been observed in Co/Al junctions [4].

Figure 2 shows the calculated result for the temperature dependence of the Hall voltages of the SJ contribution $V_{\rm H}^{\rm SJ}$ and the SS contribution $V_{\rm H}^{\rm SS}$. The $V_{\rm H}^{\rm SJ}$ and $V_{\rm H}^{\rm SS}$ normalized to the value at T_c show strong T-dependence as well as significant difference between them by the onset of superconductivity, and

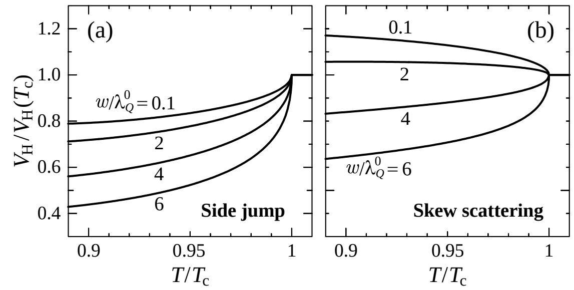

Fig. 2. Normalized Hall voltage as a function of reduced temperature for different width w of SC. (a) $V_{\rm H}^{\rm SJ}$ due to side jump, and (b) $V_{\rm H}^{\rm SS}$ due to skew scattering.

their values differ by the factor $\chi_S/2f_\Delta$. In addition, $V_{\rm H}$ depends sensitively on the width w of SC. For narrow width $w \ll \lambda_Q^0$ ($\lambda_Q^0 \sim 1\mu$m) where $G_w \sim 1$, $V_{\rm H}^{\rm SJ}$ decreases while $V_{\rm H}^{\rm SS}$ increases below T_c; For large width $w \gg \lambda_Q^0$, both $V_{\rm H}^{\rm SJ}$ and $V_{\rm H}^{\rm SS}$ decrease due to the strong decrease of G_w below T_c. If one measures the T-dependence of $V_{\rm H}$ for various width of SC, one can determine which mechanism (SJ or SS) is dominant for the Hall effect. This provides a method for distinguishing the mechanisms of the anomalous Hall effect. When the leads of a Josephson junction are connected to SC in the Hall geometry, the Hall voltage $V_{\rm H}$ induced by j_S generates the oscillation of supercurrent across the junction at a frequency $\nu = 2eV_{\rm H}/h$, thereby emitting and absorbing quanta of the frequency. If the values of $\eta_{\rm SS}$ (or $\eta_{\rm SJ}$) $\sim 10^{-2}$, $j_{\rm inj} \sim 10^5$A/cm^2, $w \sim 1\mu$m, and $P \sim 0.5$ are used, then $V_{\rm H} \sim 0.1\mu$V and $\nu \sim 10^7$ Hz.

This work is supported by a Grant-in-Aid for Scientific Research form MEXT and CREST Japan.

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