Quantum phase separation dynamics of two component Bose-Einstein Condensates

S. T. Chui, Hong Chui, Hualin Shi, W. M. Liu and Wei-Mou Zheng [a]

[a] *Bartol Research Institute, University of Delaware, Newark, DE19716, USA*

Abstract

We investigate the non-equilibrium spatial phase segregation process of a mixture of alkali Bose-Einstein condensates. A rich variety of behaviour is observed as the system parameters and compositions are changed. For compositions close to 1:1 there is a quantum spinodal decomposition with a metastable periodic density fluctuation, consistent with recent experimental results and calculations. We show that the metastability of this fluctuation depends on the relative composition of the mixture, as is suggested by recent exact solutions for this system. At extended times the density evolves into a complex pattern with domains that can oscillate, interfere, merge by period halving or by the growth of a dominant domain and reappear, or split into two (bifurcate). On top of this is a gradual separation of the densities of the two components.

Key words: dynamics, quantum, Bose-Einstein ondensates, mixtures

Recent realizations of two [1,2] and three [3] component alkali Bose-Einstein condensates (BEC's) in one trap provide us with new systems to explore the quantum physics in otherwise unachievable parameter regimes. Dramatic results were recently observed in the time dependence of the phase segregation in mixtures of Rb[1,2] and Na[3] gases. Periodic spatial structures appear at intermediate times. This phenomena was called a "quantum spinodial decomposition",[4] analogous to the classical spinodial decomposition[5], a concept describing the periodic density fluctuation in phase seperation phenomena governed by the laws of classical physics. Experimentally this structure is **metastable** in that it stays for a long time. An example of metstable structure is the soliton, which is long-lived because it is an **exact** solution of the equation of motion. Recently, we found exact solutions of this system that exhibit periodic density fluctuations. We call them straitons, in analogy to the concept of solitons. However, we can find the exact solutions **only for certain parameters and compositions**.[6] This suggests that the periodic states at intermediate time are not always metastable. To investigate some of the possibilities, we report here results of the space-time development of the density for the coupled nonlinear Schrödinger equation that describes the system. In addition to confirming our speculation of the domain of stability, we observe fasinating scenarios of patterns with many new intriguing features in the space-time density distribution of the system. We now describe our results in detail.

In this paper, we restrict our attention to cigar shape traps such as the ones studied by the MIT group so that the spatial coordinate is one dimension. We start from the time dependent Gross-Pitaevskii (G-P) or non-linear Schrödinger equations[7]

$$i\hbar\frac{\partial}{\partial t}\psi_i = \left[-\frac{\hbar^2}{2m}\nabla^2 + (U(z)-\mu_i) + \sum_j G_{ij}|\psi_j|^2\right]\psi_i. \quad (1)$$

Here $\psi_j(z,t)$, $U = 0.5m\omega^2 z^2$ with $j = 1, 2$ are the effective wave function, and the trapping potential of the condensate. The interaction between the ith and the jth condensate atoms is specified by G_{ij}. For the

[1] Corresponding author. E-mail: chui@UDel.Edu

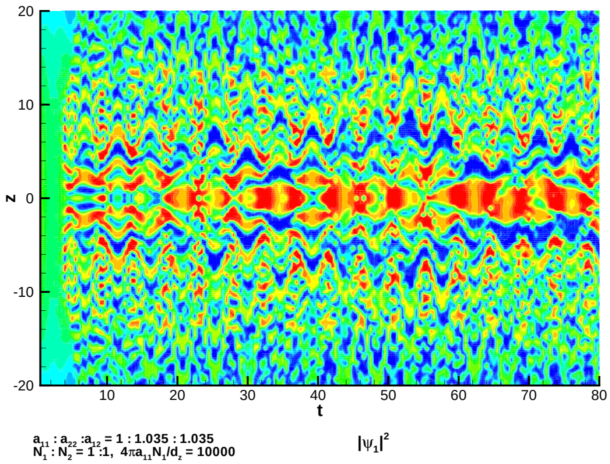

Fig. 1. The density of component 1 of a BEC condensate mixture as a function of position z and time t. The interaction parameters correspond to those of Na. Equal amounts of the two components are present.

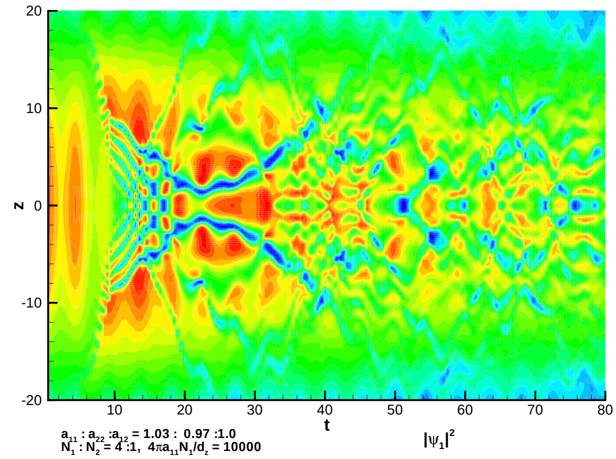

Fig. 2. The density of component 1 when the density of component two is reduced by a factor of 4 compared with the previous garph.

different spin states of ^{23}Na (^{87}Rb) $G_{11} : G_{12} : G_{22} = 1 : 1.035 : 1.035$ ($G_{11} : G_{12} : G_{22} = 1.03 : 1 : 0.97$)

The coupled nonlinear Schrodinger equation was solved numerically, the details of which is described elsewhere.[**?**] In fig. 1 we show the denisty of the first component as a function of space (in units of $2.1l_h$) and time (in units of $1/\omega$) for parameters corresponding to the experiments of the MIT group. The initial density distribution at time t=0 is given by the Thomas-Fermi approximation for the non-interacting case with $\psi_i = [(\mu_i - U)/G_{ii}]^{0.5}$, as is suggested by the experiments. We see a periodic density fluctuation for each time slice, the same as in the experiment. There is, in addition, an oscillation of the density fluctuation with a time scale of the order of $5/\omega$. The sum of the densities of components 1 and 2 is smooth, similar to that reported experimentally by the JILA group[2] This comes about because the G_{ij}s are close to each other in value.[10] There is an overall oscillation of this total density as a function of time.

To investigate the question of metastability as a function of composition, we have carried out a calculation where the density of component 2 is reduced by a factor of 4. The density for component 1 is shown in Fig. 2. There is a spatially-periodic density fluctuation at short time. As expected, this structure is short lived. In its stead, a very intricate and fascinating pattern develops as time progresses. The spatial separation between the two components is less than the 1:1 case. There seems to be an interference pattern close to the center from t=10 to t=20.

In conclusion, we have examined the quantum phase separation of mixtures of Bose-Einstein condensates. We found a spatially periodic structure at short time, consistent with results from a linear stability analysis. The spatially periodic structure is metastable **only** for compositions close to 1:1 with a small periodicty superimposed on top. For other system parameters, we observe fasinating scenarios of patterns with many new intriguing features that we hope will stimulate further investigations.

This work was supported in part by a grant from NASA (NAG8-1427)

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