

# Singular and nonsingular vortices in high-temperature superconductors

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## Abstract

In this paper we focus on the comparison of equilibrium and transport properties of singular and nonsingular vortex structures in high-temperature compounds. Using the time-dependent Ginzburg-Landau theory we study the dynamics of vortex structures in superconductors with  $(d+s)$ -wave pairing. We calculate the angular dependent correction to the viscosity tensor of a singular flux line, which appears due to the admixture of subdominant  $s$ -wave order parameter component in the vortex core. The second order phase transition between singular and nonsingular vortices in high-temperature superconductors was simulated within time-dependent Ginzburg-Landau equations.

*Key words:* mixed state, unconventional pairing, high-temperature superconductors

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In the last decade a number of experiments provided evidence of nontrivial pairing with coexisting  $s$ -wave and  $d$ -wave (or  $d_{x^2-y^2}$ -wave and  $d_{xy}$ -wave) order parameter (OP) components in high-temperature superconductors. It is obvious that interaction between different OP components in unconventional superconductors provides the rich physical picture both in equilibrium and dynamical regimes. In particular, the existence of additional OP component has an influence on the various characteristic of mixed state in high-temperature superconductors, e.g., critical magnetic fields, dynamical properties etc.

In this paper we study the dynamical features of flux lines in  $(d+s)$ -wave superconductors using the time-dependent Ginzburg-Landau (GL) theory. One can write the GL free energy functional of anisotropic high-temperature superconductor with  $(d_{x^2-y^2}+s)$ -wave pairing in the following form:

$$F = \int (f_s + f_d + f_{sd}) \, dV, \quad (1)$$

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$$\begin{aligned} f_q &= a_q |\Psi_q|^2 + \frac{b_q}{2} |\Psi_q|^4 + K_q |(\mathbf{\Pi}_{||} + \gamma_q \Pi_z \mathbf{z}_0) \Psi_q|^2 \\ f_{sd} &= \beta_1 |\Psi_d|^2 |\Psi_s|^2 + \frac{\beta_2}{2} (\Psi_d^2 \Psi_s^{*2} + \Psi_d^{*2} \Psi_s^2) \\ &+ K_{sd} [(\Pi_x^* \Psi_s^* \Pi_x \Psi_d - \Pi_y^* \Psi_s^* \Pi_y \Psi_d) + c.c.] \end{aligned}$$

where  $\Psi_d(\mathbf{r})$  and  $\Psi_s(\mathbf{r})$  are the components of OP,  $\Pi_{||} = \nabla_{\mathbf{r}} - i \frac{2\pi}{\Phi_0} \mathbf{A}_{||}$ ,  $\Pi_z = \nabla_z - i \frac{2\pi}{\Phi_0} A_z$ ,  $\mathbf{r} = (x, y)$ ,  $\Phi_0$  is the flux quantum,  $\mathbf{A} = (\mathbf{A}_{||}, A_z)$ ,  $\mathbf{H} = \text{curl} \mathbf{A}$ ,  $a_s = \alpha_s (T - T_{cs})$ ,  $a_d = \alpha_d (T - T_{cd})$ , and  $\gamma_s$ ,  $\gamma_d$  are the anisotropy parameters for  $s$ - and  $d$ -wave OP components. We choose  $x, y, z$  lying along the  $a, b$  and  $c$  crystallographic axes, respectively.

The time-dependent GL equations are

$$\begin{aligned} \eta_q \left( \hbar \frac{\partial}{\partial t} + 2ie\Phi \right) \Psi_q &= -\frac{\delta F}{\delta \Psi_q^*}, \\ \text{div}((\mathbf{j}_s - \sigma_n \nabla \Phi) &= 0, \\ \mathbf{j}_s &= -c \frac{\delta F}{\delta \mathbf{A}}, \end{aligned} \quad (2)$$

where  $\delta F / \delta \Psi_q^*$  is variational derivative of free energy (1) and  $q = s, d$ .

There are two types of vortices which can be realized in  $(d+s)$ -wave superconductors: (i) singular vortices

[1–4] which have at least one point where the superconducting gap is zero and (ii) nonsingular vortices [5] where the gap is nonzero everywhere in the vortex core.

When the parameters of GL theory are chosen so that there is pure  $d$ -wave homogeneous state without magnetic field, the structure of singular flux line in ( $d+s$ )-wave superconductors exhibits a fourfold symmetry of arrangement of  $s$ -wave unit vortices around the  $d$ -wave unit vortex for the magnetic field applied along the  $c$ -axis (see e.g. [1,2]). As magnetic field tilts from the  $c$ -axis, the fourfold symmetry of a flux line turns to the twofold one due to anisotropy of mass tensor [3,4].

We have calculated the angular dependent correction to viscosity tensor  $\hat{\eta}$  of tilted singular flux line using the time-dependent GL equations (2). It is obvious that the internal structure of the flux line affects the viscosity tensor of flux line. An additional term of  $\hat{\eta}$ , which depends on magnetic field direction is proportional to the additional term of the loss function which is due to the presence of a  $s$ -wave component in the vicinity of flux line core.

Let us introduce the unit vector  $\mathbf{n}$  along a vortex line:  $\mathbf{n} = (\sin \theta \cos \alpha, \sin \theta \sin \alpha, \cos \theta)$  and transform the system coordinate so, that the new  $\tilde{z}$ -axis will be parallel with  $\mathbf{n}$

$$\tilde{x} = [x \cos \alpha + y \sin \alpha] \cos \tilde{\theta} - \gamma_d z \sin \tilde{\theta}, \quad (3)$$

$$\tilde{y} = -x \sin \alpha + y \cos \alpha,$$

$$\tilde{z} = [x \cos \alpha + y \sin \alpha] \sin \tilde{\theta} + \gamma_d z \cos \tilde{\theta},$$

where  $\tilde{\theta} = \tan^{-1}(\gamma_d^{-1} \tan \theta)$ .

We study the stationary flux flow regime when flux line moves with velocity composing the angle  $\chi$  with the  $\tilde{x}$  axis. We assume that the vortex velocity is rather small that allows us to use a perturbation theory based on static solution of vortex structure. Also we assume that temperature is close to  $T_{cd}$ , therefore we can use perturbation theory with small parameter  $a_d(T)/a_s(T) \ll 1$ . In this case, the induced  $s$ -wave OP component is much smaller than  $d$ -wave one.

The additional term in the viscosity tensor, which is connected with the  $s$ -wave OP component, takes the following diagonal form in coordinate system (3)

$$\begin{aligned} \hat{\eta}_1 &= \frac{9\pi\hbar\eta_d}{2b_d} \frac{|a_d(T)|K_{sd}^2}{a_s(T)K_d} g(\alpha, \theta, \chi) \\ &\quad \times \begin{pmatrix} \cos \theta \sqrt{1 + \gamma_d^2 \tan^2 \theta} & 0 \\ 0 & 1 \end{pmatrix}, \\ g &= \left[ \cos^2 2\alpha \left( \sin^4 \tilde{\theta} + \frac{7}{2} (1 + \cos^2 \tilde{\theta})^2 \right) \right. \\ &\quad \left. + 14 \sin^2 2\alpha \cos^2 \tilde{\theta} \right] \\ &\quad \times \left( \left( \frac{1 - \gamma^2 \tan^2 \theta}{1 + \gamma^2 \tan^2 \theta} \right)^2 \cos^2 \chi + \sin^2 \chi \right), \end{aligned}$$

The temperature dependence of fourfold  $\alpha$ -dependent correction to  $\hat{\eta}$  has a positive curvature with decreasing  $T$ , that reflects the increasing of admixture of  $s$ -wave OP component.

Using the time-dependent GL simulation we also have studied the possible formation of nonsingular vortices [5] with non-zero  $s$ -wave OP component in the center of flux line. This process appears to be possible under the certain conditions due to nonlinear interaction between  $s$ - and  $d$ -wave OP components. The nucleation of  $s$ -wave OP component with winding number  $N = 0$  results in the mutual shift of  $s$ -wave and  $d$ -wave unit vortices so that a certain vector lying in the  $ab$  plane can be associated with the nonsingular vortex. Such an effect results in the anisotropy of transport properties of nonsingular vortices in the  $ab$  plane in contrary to the singular vortices. The appearance of strong  $s$ -wave nucleus in the nonsingular vortex provides an additional losses in flux flow regime, i.e., the viscosity of nonsingular vortex should be more larger than the viscosity of singular one.

Generation of a subdominant OP component may cause a partial reduction of anisotropy of quasiparticle density of states (DOS) in the  $d$ -wave vortex. The resulting gap function  $\Delta(\mathbf{k}, \mathbf{r}) = \Psi_s(\mathbf{r})\Delta_s(k) + \Psi_d(\mathbf{r})\Delta_d(\mathbf{k})$  appears to be nodeless in the core of nonsingular vortex and, probably, form a DOS with two peaks.

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## References

- [1] A.J. Berlinsky, A.L. Fetter, M. Franz, C. Kallin, P.I. Soininen, Phys. Rev. Lett. **75**, (1995) 2200; M. Franz C. Kallin, P.I. Soininen, A.J. Berlinsky, A.L. Fetter, Phys. Rev. B **53**, (1996) 5795; R. Heeb, A. van Otterlo, M. Sigrist, G. Blatter, Phys. Rev. B **54**, (1996) 9385.
- [2] D. Chang, C.-Y. Mou, B. Rosenstein, and C.L. Wu, Phys. Rev. B **57**, (1998) 7955; Q. Li, Z.D. Wang, Q.-H. Wang, Phys. Rev. B **60**, (1999) 14577; Q.-H. Wang, Z.D. Wang, Q. Li, Phys. Rev. B **60**, (1999) 15364.
- [3] J.J. Vicente Alvarez, C.A. Balseiro, G.C. Buscaglia Phys. Rev. B **58**, (1998) 11181.
- [4] A.S. Bystrov, A.S. Mel'nikov, D.A. Ryzhov, J. Phys.: Cond. Matt. **13**, (2001) 6005.
- [5] A.S. Mel'nikov, I.M. Nefedov, D.A. Ryzhov, I.A. Shereshevskii, P.P. Vysheslavtsev, Phys. Rev. B **62**, (2000) 11820.