Thermal Gravity-Driven Convection of the Low Temperature Fluids in Enclosures

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Abstract

Numerical simulation of the thermal gravity-driven convection with bottom heating (the Rayleigh-Benard configuration) in near-critical ^3He is performed. The 2D numerical code designed on the basis of the full Navier-Stokes equations with the van der Waals equation of state is used. We have found the calibration forms relating dimensionless parameters in governing equations with "real" criteria of similarity of a near-critical fluid. A comparison with the recent experimental data on the onset of convection shows a good agreement between the model and the physical fluids.

Key words: thermodynamic critical point, heat transfer, convection onset, numerical simulation

In the vicinity of the thermodynamic critical point, media named as near-critical fluids display specific thermodynamic and kinetic properties, namely, an asymptotic discrepancy of the specific heat at constant pressure, of the coefficient of isothermal compressibility, and the slowing down of the thermal diffusion. These abnormal properties lead to specific heat transfer and interesting hydrodynamic effects.

Near-critical dynamic problems are solved numerically. The full Navier-Stokes equations describing compressible viscous media, equation of energy for a non-perfect gas, and the van der Waals equation of state are considered [1]. The coefficient of heat conductivity λ, increasing near the critical point, is described (in dimensionless form) by the power-law relation

$$\lambda = 1 + \Lambda \cdot \varepsilon^{-\psi} \qquad (1)$$

Here, $\varepsilon = (T - T_c)/T_c$ is the temperature distance from the critical point (or reduced temperature), T_c is the critical temperature. On the basis of the model, the novel 2D-numerical code was designed.

The dimensionless set of equations includes the Rayleigh Ra and Prandtl Pr numbers. With approach to the critical point the criteria of similarity are known to tend to infinity, while the values Ra and Pr remain the same. To describe the near-critical convection completely, we consider the "real" Rayleigh Ra_r and the "real" Prandtl Pr_r numbers, which take the real physical properties in the vicinity of the critical point into account. They are signed by subscript "r" and expressed as (near critical isochore) [2]

$$Ra_r = \frac{2}{3}\varepsilon^{-1}\left(\frac{1}{\gamma_0} + \frac{\gamma_0 - 1}{\gamma_0}\cdot\frac{1+\varepsilon}{\varepsilon}\right)\frac{1}{\lambda}Ra,$$

$$Pr_r = \left(\frac{1}{\gamma_0} + \frac{\gamma_0 - 1}{\gamma_0}\cdot\frac{1+\varepsilon}{\varepsilon}\right)\frac{1}{\lambda}Pr \qquad (2)$$

Here, γ_0 is the ratio of specific heats for a perfect gas. The parameters Ra_r and Pr_r diverging asymptotically, as $\varepsilon \to 0$: $Ra_r/Ra \sim \varepsilon^{\psi-2} \to \infty$, $Pr_r/Pr \sim \varepsilon^{\psi-1} \to \infty$ $(\psi < 1)$.

The critical point in ^3He is characterized by the temperature T_c = 3,3189 K, the density ρ_c = 0,0414 g/cm^3, and the pressure P_c = 0,117 MPa. We used the experimental data on ^3He [3, 4] and received the

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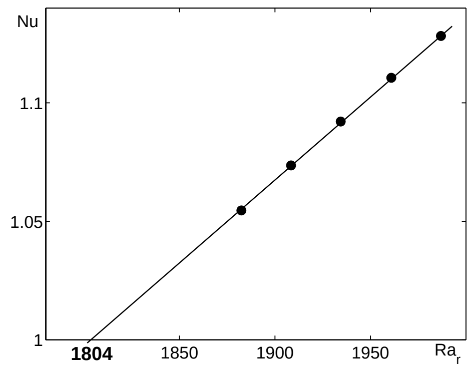

Fig. 1. The Nusselt number Nu versus the "real" Rayleigh number Ra_r (marks) and fitting linear curve (solid line)

constants $\Lambda = 0{,}0149$, $\psi = 0{,}645$ in (1). We also evaluated the values of the Rayleigh number Ra_e including measured physical quantities at different ε. The experimental Prandtl numbers Pr_e were known from [5]. To ensure the agreement between convective processes in physical and model fluids in the certain vicinity of the critical point, we need to choose the values of Ra and Pr in governing equations giving the same Ra_r and Pr_r (calculated from (2)) as Ra_e and Pr_e in experiments.

The steady-state Rayleigh-Benard convection in helium close to experiments, which carried out in a flat cell (height - $1mm$, diameter - $57mm$) [3, 4] is studied. The temperature of the top plate was kept at a fixed value, the bottom plate was heated very slowly ensuring a quasi-steady state at every instant.

We consider a part of a whole cell presenting a square cavity (height and length equal to $1mm$) assuming slip and adiabatic conditions at vertical boundaries. The temperature at the upper surface is $0{,}33K$ above critical, at the lower surface is larger by some magnitude θ. The parameters $\varepsilon = 0{,}1$, $Pr = 0{,}814$, $\gamma_0 = 1{,}67$ are used. The values $\theta = 14{,}4 - 15{,}2\ mK$ and, consequently, $Ra = 60{,}0 - 63{,}4$ are varied giving a set of stationary solutions.

The Rayleigh number $Ra_{r-onset}$ characterising the convection onset is received by extrapolating data on the Nusselt number Nu to the value $Nu = 1$, where convection is absent. The Ra_r dependence of Nu is supposed to be a linear form that gives the threshold value $Ra_{r-onset} = 1804$ (see Fig. 1). The obtained results are in a good agreement with the experimental data [3, 4] and the theoretical consideration [6]. As predicted earlier, $Ra_{r-onset}$ is slightly larger then the classical magnitude in incompressible fluids (equals to 1708) because of the nonzero adiabatic temperature gradient.

Tending to the critical point, the influence of compressibility and adiabatic temperature gradient is enhanced leading to dominating role of the Schwarzchild criterion [6]. The stability problems were discussed in [7] as well. In present study, we determined the convection onset only at one sufficiently large reduced temperature $\varepsilon = 0{,}1$. Experiments [3, 4] have shown, that the role of the Schwarzchild criterion is not significant at that distance from the critical point. The influence of compressibility is not involved into consideration here, but it should be taken into account for analysis of the closest near-critical neighborhood. By the way, the applied mathematical model, effective 2D numerical code, and developed approach to recognizing obtained results provide a possibility of simulating and analyzing the complex dynamic problems in near-critical fluids close to experimental conditions.

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