Turbulence at low temperatures

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Abstract

Cryogenic helium is of significant value in generating, and studying the highest possible Reynolds and Rayleigh number flows under controlled laboratory conditions, primarily due to its extremely low value of kinematic viscosity. We consider here critical helium gas and the two liquid phases, helium I and helium II. Such flows are already being generated and studied using suitable cryogenic equipment. We outline the current experiments and existing proposals for future studies that include critical helium gas and liquid helium I and II.

Key words: quantum turbulence; convection; Reynolds number; Rayleigh number

1. Introduction

Turbulent flows occur commonly in nature and play an essential role on all scales, from galactic formation to the circulation of blood within our bodies. Measures of turbulent intensity are given by the Reynolds number $Re = UL/\nu$ for essentially isothermal flows with characteristic length scale L and velocity U, and the Rayleigh number $Ra = (g\alpha\Delta TL^3)/\nu\kappa$ for thermally driven flows in a gravitational field g, where ν α and κ are respectively the kinematic viscosity, thermal expansion coefficient and thermal diffusivity, and ΔT is the temperature difference in the direction of g. Note that most natural turbulent phenomena occur on large scales L and are characterized by large values of these numbers. For example in the sun Rayleigh numbers of order 10^{21} and Reynolds numbers of order10^{13} are observed; in the atmosphere the corresponding numbers are 10^{17} and 10^9. The Reynolds number of a nuclear submarine at full throttle can reach 10^9.

It is well known that despite a long period of intensive research a general theory of turbulence is still lacking and computer simulations based on the Navier-Stokes equation are limited to moderate Re numbers of order several hundred. Since controlled laboratory experiments are constrained in L, it is desirable to study such situations using a working fluid possessing the smallest possible ν.

Helium offers three cryogenic fluids of interest. The first is critical helium gas, a heavy gas of helium near the critical point whose temperature is 5.20K and whose pressure is 2.26 atm. Dynamically it is a Navier-Stokes fluid. By that we mean the flow is accurately characterized by the Navier-Stokes equation, and is a Newtonian fluid, that is, the stress tensor is linearly related to the rate of strain tensor. Critical helium gas has properties strongly dependent upon the temperature and pressure.

It is important to point out that helium is not only the fluid of lowest viscosity, it is able to reach a number of remarkable extremes in various ways. For example, in thermal convection using critical helium gas the Rayleigh number can be extremely large due to the ratio of fluid properties $\alpha/\kappa\nu$ appearing in the definition, which is some ten million times larger than for water.

This is necessarily a brief summary of progress to date. Three volumes of conference proceedings have appeared covering this material in considerable detail. They are contained in references [1–3].

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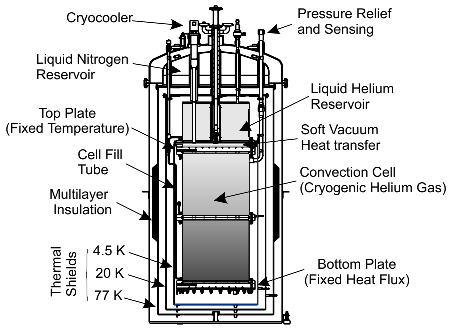

Fig. 1. Sketch of the university of Oregon 1 meter convection cell.

2. Turbulent thermal convection experiment

Rayleigh - B 饅’enard convection occurs in a horizontal layer of fluid of thickness L confined between perfectly conducting top and bottom surfaces. The bottom surface is heated and the top cooled in such a way that a steady temperature difference ΔT is maintained between the surfaces. In the Boussinesq approximation variations of all fluid properties other than the density are ignored completely. Variations of the density are ignored except insofar as they give rise to a gravitational force. ¿From the dimensional analysis of the equations of motion it follows that the description of the thermal convection depends upon two dimensionless parameters, the Rayleigh number defined above and on the Prandtl number $Pr = \nu/\kappa$. In particular, the convective heat transfer, H, can be expressed by the Nusselt number $Nu = H/H_0 = f(Ra, Pr)$,where $H_0 = \kappa\Delta T/L$ is the heat transport due to conduction only. The functional dependence above depends on boundary conditions, in particular on the aspect ratio $\Gamma = D/L$, where D is the horizontal size of the convection cell. Depending on the aspect ratio, heat is typically transferred by conduction alone for $Ra < 2000$, i.e. $Nu = 1$. For higher Ra there are successive regions of steady convection, oscillatory convection, chaos, transition to turbulence, and regions sometimes called soft and hard turbulence that are identified by power-law scaling relations $Nu \propto Ra^m$ (assuming Pr stays approximately fixed). Of particular interest (for application in astrophysics, geophysics, oceanography, meteorology, etc.) is an asymptotic form of this scaling law for very high Ra.

The first study of thermal convection in helium I was due to Ahlers [4]. While working on a somewhat unrelated matter, Brian Pippard came to realize the potential of critical helium gas for a Rayleigh-Bénard experiment. His student, the late David Threlfall, used a small cylindrical cell with aspect ratio $D/L = 2.5$ (D=48.4 mm) and investigated a range of Ra from 60 up to 2×10^9. From the upper four decades of Ra he obtained the scaling relation $Nu \propto Ra^m \cong 0.173Ra^{0.28}$ [5]. This research was continued and extended by Libchaber's group in Chicago: Sano et al [6], Wu and Libchaber [7]. Wu [8] reported results on Rayleigh Bénard experiments with cylindrical experimental cells of aspect ratio 0.5, 1 and 6.7 for Ra up to 10^{14}. They observed large-scale coherent flow, and from the temperature fluctuation spectra were able to distinguish a transition between so-called ”soft” and ”hard” turbulence. The transition between those two regimes was reported to be around $Ra \approx 10^8$, but the scaling relation between Nu and Ra did not appreciably change. They found that for Ra above about 10^8 Nu scales with Ra with the power close to $m \simeq 2/7$. Clearly, there was a need for experiments in which higher Ra could be achieved. Therefore a much larger cell, 1 m high and 0.5 m in diameter, was designed and constructed at Oregon using critical helium gas ($4.3K < T < 6K, 0.1mbar <$P$< 3bar$) as a working fluid [9]. This apparatus has the capability to span up to eleven orders of magnitude of the control parameter, Ra, (up to 10^{17}), the highest Ra ever reached in the laboratory. Results are shown in Fig.2. Such a large range of Ra in a single experiment is achieved by changing the density and mean temperature of the working fluid together with the temperature difference between the lower and upper copper plates. Our experiments [9] show no signs of any transition into the predicted ultimate $Nu - Ra$ scaling ($m = 0.5$), at least up to $Ra = 10^{17}$. Our experiments showed the existence of organized flow features observed earlier by others such as plumes, jets, and a large scale circulation known as the ”wind”. The size of this circulation is of the order of the size of the convection cell. We have studied this feature experimentally in an apparatus of aspect ratio unity, in which the highest available Rayleigh number is of order 10^{16}. Over a wide range of time scales greater than its characteristic turnover time, the wind velocity exhibits occasional and irregular reversals without a change in magnitude [10]. Space limits discussion of many other aspects of the observations such as spectra of fluctuations and structure functions. Interested readers are directed to reference [11]. Although the Oregon cell was designed to reach the highest Ra yet in a controlled laboratory experiment (under Boussinesq conditions) it is the prototype of a much larger experiment designed to study ultra-high Ra flows involved in large scale natural phenomena. A conceptual design for a 10 m cell was presented some time ago [12]. Experience gained with the 1 m cell will be invaluable in designing the 10 m cell, which will hopefully be located at Brookhaven National Laboratory, and will be capable of reaching Rayleigh numbers in excess of 10^{20} .

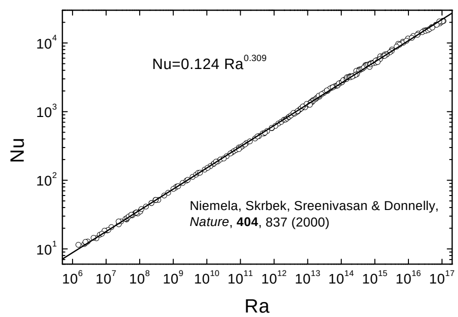

Fig. 2. Log-log plot of the Nusselt number versus Rayleigh number from the Oregon convection cell. The slope is 0.309 over the entire range.

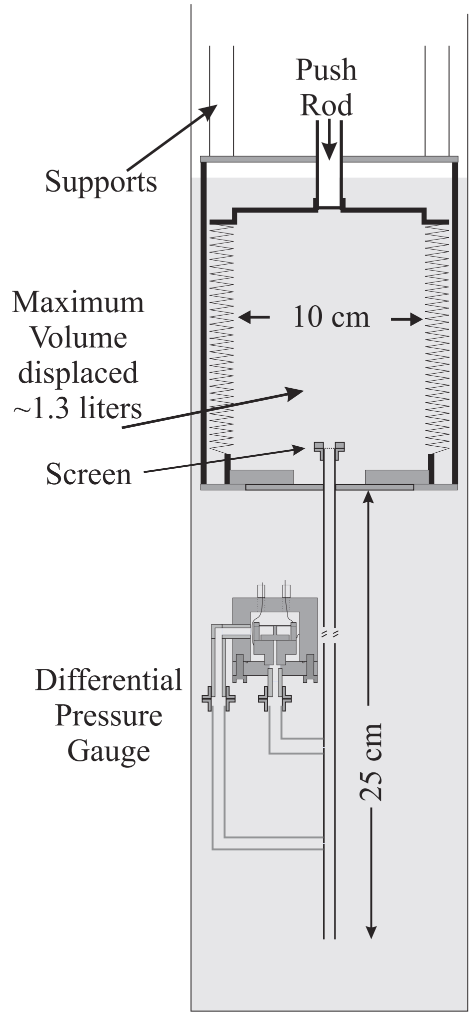

Fig. 3. Sketch of the pipeflow apparatus.

3. Wide-Range Reynolds number pipe flow using liquid helium and various gases

We describe here the development of low temperature measurement tools and the use of helium I to reproduce standard friction factor measurements in fully developed laminar and turbulent flow in a pipe. We sought to make accurate measurements spanning a broad range of Reynolds numbers in a very small apparatus. Using a flow pipe only 28 cm long and 0.4672 cm in diameter we were able to span a Reynolds number range from 10 to 10^6 by taking advantage of both helium I at 4.2 K and gaseous helium, nitrogen, oxygen, carbon dioxide, and sulfur hexafluoride. This is the largest range of Reynolds numbers ever covered in a single apparatus. For many turbulence experiments, the range of Reynolds numbers measured may be every bit as important as reaching the highest Reynolds number. The flow through the pipe is generated by a controlled compression of a metal bellows attached to the mouth of the pipe. The pressure gradient generated by the flow is measured near the exit of the pipe with two static pressure taps a distance L apart. From the mean velocity and the pressure gradient across the taps, we can determine the friction factor λ given by $\lambda = 2D\Delta P/L\rho V^2$ where D is the diameter of the pipe, ΔP is the pressure drop over the distance L, the fluid density is ρ, and the mean fluid velocity is V. Results are shown in Fig.2.

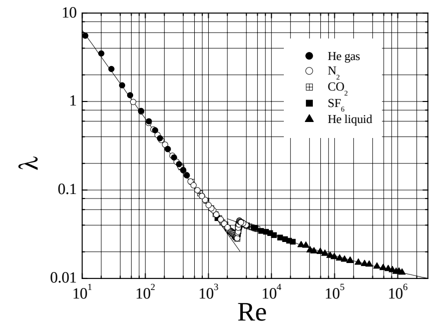

Fig. 4. Friction factor as a function of Reynolds number. The use of different fluids, including cryogenic helium gas and helium I, produces the widest range of Reynolds numbers ever achieved in a pipe flow experiment. The solid line represents the accepted results.

4. Quantum Turbulence

4.1. *Counterflow Turbulence*

Counterflow turbulence has been studied in helium II since the late 1930's. In the steady state the experiment usually consists of a channel of some sort heated at one end and cooled at the other. The superfluid can be thought of as having no entropy; it flows to the heater, picks up entropy to become normal fluid, and counterflows against the incoming superfluid in such a way that there is no net mass flow. Obviously this peculiar situation has no classical counterpart. For low heat fluxes the only dissipation is the friction of the normal fluid on the walls of the channel. At higher heat fluxes there arises what appears to be a force of mutual friction between the fluids which was described qualitatively by Gorter and Mellink using a term cubic in the relative velocity between the two fluids. The microscopic origin of this mutual friction was later identified by Vinen as arising from a tangle of quantized vortex lines. Vinen was able to make a theory of this mutual friction which essentially stands unchanged today [13]. Hundreds of papers have been published on counterflow turbulence, and this vast literature has been covered by a number of review articles, particularly Tough [14], Donnelly and Swanson [15], Donnelly [16]. Donnelly and Swanson introduced the name 'quantum turbulence' for this phenomenon, principally on the grounds that if Planck's constant were made zero, the effect would disappear completely. More recently Vinen has argued that under some circumstances, quantum effects will dominate certain turbulent flows, and a review of the field has been prepared recently by Vinen and Niemela. [17].

4.2. *Towed Grid Turbulence*

Numerous investigators have studied grid turbulence, regarded as nearly homogeneous and isotropic, as it decays downstream in a wind tunnel. It is desirable to study the decay of high Reynolds number turbulence possessing a well-developed inertial range, such as can be provided by using helium II. The extremely low viscosity of helium II, lowest of all known substances, allows such a goal to be reached in an apparatus of small size and under controlled laboratory conditions. We use a technique reported in [2,3], where turbulence is created by towing a grid with velocity V_g through a stationary sample of helium II. A wide range of mesh Reynolds numbers ($2 \times 10^3 \leq Re_M = V_g M\rho/\mu \leq 2 \times 10^5$) can be achieved easily in a small $1 \times 1 \times 29$ cm^3 channel using a grid with mesh size M=0.167 cm. Here μ is the dynamic viscosity and ρ the density of helium II. The probe for this flow is second sound attenuation, which measures the quantized vortex line length per unit volume L. In the framework of the phenomenological two-fluid model helium II is described as consisting of two independent fluids: the inviscid superfluid of density ρ_s , and the normal fluid of dynamic viscosity μ and density ρ_n, where $\rho = \rho_s + \rho_n$. In turbulent flow, however, the presence of quantized vortices couples the two fluids together (at least for scales larger than the intervortex spacing), via mutual friction [3]. Note that we are measuring quantized vortices in the superfluid and nothing in the normal fluid. In the temperature range

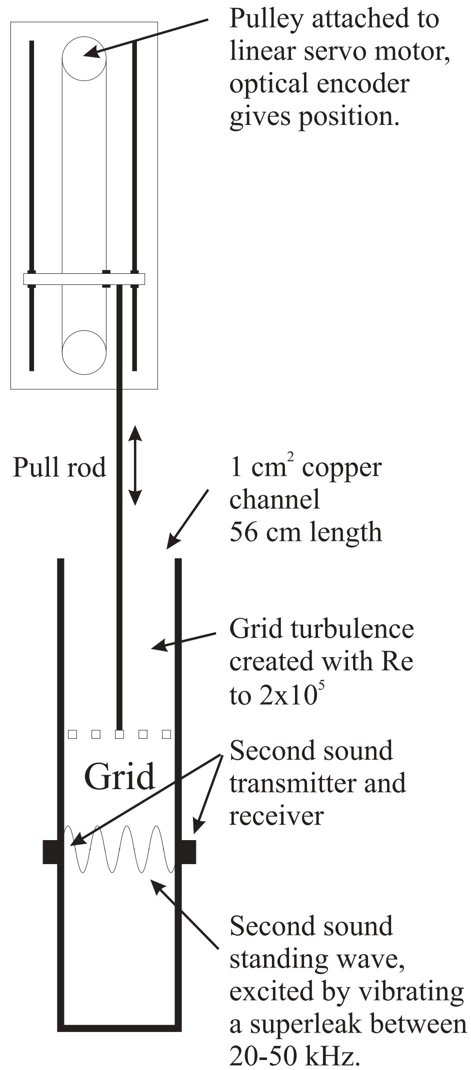

Fig. 5. Sketch of the towed grid apparatus. Details and scale are not shown.

covered in the present study, turbulent helium II flow resembles a classical flow possessing an effective kinematic viscosity of order $\nu = \mu/\rho$ [3]. Here the vorticity is *defined* as $\omega = \kappa L$, where κ is the quantum of circulation. In particular, the usual relationship relating turbulent energy dissipation per unit volume to the mean square vorticity applies: $\epsilon = \nu\omega^2$. This technique allows measurements of vorticity from 10^4 Hz to 0.01 Hz. It is this unprecedented range of observed values of vorticity and flexibility in Re_M that allows detection of several distinctly different regimes of the decay and makes our system unique in the study of turbulence. Note that $\omega \propto t^{-3/2}$ for most of the decay so that the relationship $\epsilon = \nu\omega^2$ implies we are observing over eight orders of decaying turbulent energy, clearly an impossible goal for any conventional labora-

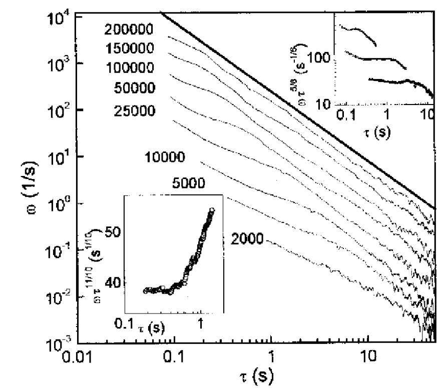

Fig. 6. The decaying superfluid vorticity measured at T=1.3 K for the indicated Re_M

tory experiment (reaching such a dynamic range in a classical wind tunnel would require its test section to be more than 1000 km long). We use this method in a temperature range $1.2K < T < 2K$, for which ρ_n/ρ varies by more than factor of ten. Despite this large variation in the normal fluid fraction, we observe no appreciable difference in the turbulent decay curves obtained over this wide temperature range. Results [18] are shown as a sequence of different decay laws in Fig. 3. Each curve represents an average of three individual decays. As the decay curves tend to collapse on the universal curve, we shifted them for clarity by a factor of two downwards, the uppermost remaining unchanged. The early part of the vorticity decay displays a power law with exponent -11/10 (see left inset, showing normalized data for $Re_M = 10^4$) and later -5/6 (see right inset, showing normalized data for $Re_M = 1.5 \times 10^5, 2.5 \times 10^4$, and 5×10^3). After saturation (the regime where the energy containing eddies have reached the size of the channel) typically several orders of magnitude of decaying vorticity closely follow the power law with exponent -3/2, represented by the thick solid line. These three stages of decay have classical counterparts. It has been astonishing to us that classical models should account for so much of the decay of grid turbulence. A fourth, final stage of decay is seen, which is exponential in form. However at this stage there are hardly any vortex lines left in the channel, so any continuum model one might make is likely to be uncertain at best. On the whole, our findings suggest a deep similarity between conventional and superfluid turbulence. The underlying quantum physics has been discussed elsewhere by Vinen and Niemela [17].

5. Pipe flow results in helium II

Turbulence generated by conventional means (as distinct from counterflow turbulence) can be divided into two groups: flows in which boundary layers play a critical role such as flow through a pipe or over a bluff object, and those in which boundary layers do not play a critical role such as grid turbulence discussed above and swirling flow [19] . This distinction is critical for understanding superfluid turbulence for the following reason. In flows where boundary layers do not play a critical role, such as grid turbulence, the large scale motions are largely independent of the smaller scale motions. Since viscosity becomes important only at the smallest scale in the flow, turbulence at the larger scales can be treated by inviscid models. In classical boundary layer phenomena, on the other hand, viscosity plays a significant role in mean quantities such as velocity profile and pressure drop. Thus classical boundary layer turbulence cannot be treated by inviscid models. Helium II turbulent boundary layer flows will differ from their classical counterparts because of the existence of two fluids, one of which has no viscosity. Perhaps more importantly we do not know what the superfluid boundary conditions are at a solid surface when vortex lines are present. Certainly they will depend on the roughness of the surface We have recently completed some systematic measurements of pipe flow in helium II in our own laboratory, which are being prepared for publication. We define the Reynolds number in pipe flow of helium II as $Re = VD/\mu$, where V is the mean flow velocity, D is the diameter of the pipe, ρ is the total density of the fluid and μ is the normal fluid viscosity. We define the kinematic viscosity here as $\nu = \mu/\rho$, but it is always a question as to whether this can be correct at all times in turbulent flows. In contrast to towed grid turbulence our results show a strong temperature dependence in helium II, for reasons alluded to above. The structure of boundary layers in classical turbulence theory is a formidably complicated affair, and there is no reason to think the situation in helium II will be any simpler. Much work remains to be done to understand these spectacular results.

6. Conclusions

The results presented here are but a tiny part of what is becoming a large field of investigation, with unexpected ramifications for both classical turbulence investigations and for the understanding of quantum turbulence. There are other major directions which I have not had the space to cover. These include the area of high Reynolds number wind tunnel testing and tow tank testing in engineering. These matters are treated in greater detail in references [1,2]. I think it is fair to say that the classical turbulence community now accepts and increasingly understands quantum turbulence as an intellectually challenging field with exciting questions for all to work on. Starting with the auspicious investigations of Joe Vinen in the 1950's, turbulence at low temperatures has truly arrived as a major subject in its own.

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