Magnetic relaxation in superconductors with rotating flux lines

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Abstract

Magnetic relaxation of the critical state generated by a flux-line cutting process is theoretically investigated for a superconducting plate. It is shown that study of the magnetic flux decay measured in two mutually perpendicular directions in the plane of the plate allows one to extract the effective height of the activation barrier against flux-line cutting from experimental data.

Key words: flux-line pinning; flux-line cutting; critical state in type-II superconductors; magnetic flux creep

Experimental data on magnetic relaxation in type-II superconductors enable one to estimate the so-called effective depth of a flux-pinning well, U_0, using the expression [1] for the magnetic relaxation rate $R \equiv d|M|/d\ln t$:

$$R = -\frac{T}{U_0}|M|, \quad (1)$$

where M is the magnetic moment of the sample, T is its temperature, and t is the time. Strictly speaking, for this formula to be valid, the current in any point of the superconductor has to flow perpendicular to the flux lines. For plates, strips, disks and cylinders this property is caused by the symmetry of the problem. However, in real samples of less symmetric shapes, adjacent flux lines may be slightly rotated relative to each other. This rotation generates a component of the current along the magnetic induction $\mathbf{B}$. In other words, the current density $\mathbf{j}$ is no longer perpendicular to the flux-lines. Such a rotation may exist even in samples of sufficiently simple shape. For example, it occurs in the partly penetrated critical states of flat isotropic superconductors of elliptic or rectangular shape [2,3]. A rotation can be also created in a plate in special experiments [4].

The rotation of flux lines in superconductors can lead to their mutual cutting [5,6]. Flux line cutting occurs when the gradient of the angle defining the direction of $\mathbf{B}$ in the sample exceeds some critical value k_c or, equivalently, the component of current density parallel to $\mathbf{B}$, $j_\parallel$, exceeds the longitudinal critical current density $j_{c\parallel}$. In this situation a vortex [7] or a vortex array [8] becomes unstable with respect to a helical distortion, and the growth of this distortion leads to flux-line cutting. As a result, a so-called general critical state [6,9] is established in the superconductor when both $j_\parallel$ and the component of $\mathbf{j}$ normal to the magnetic induction, $j_\perp$, are equal to their critical values $j_{c\parallel}$ and $j_{c\perp}$, respectively. The *relaxation* of just this state is the main subject of our paper. It turns out that apart from U_0, one more parameter U_1 is required for describing this relaxation. This U_1 is the effective height of the barrier which prevents flux-line cutting in superconductors. Knowledge of U_0 and U_1 enables one to describe the magnetic relaxation in samples of arbitrary shape and with any magnetic prehistory. Our results for the magnetic relaxation in a plate provide the possibility to obtain U_1 from experiments.

We consider a geometry which is similar to that used in the experiments of LeBlanc et al. [4]: A superconducting plate of thickness d is placed in an external homogeneous magnetic field H parallel to its plane; then the field is rotated in the plane by an angle ζ (or, equivalently, the sample by an angle $-\zeta$) without changing its magnitude, and eventually the direction and the magnitude of the field are fixed and remain constant;

this moment of time is taken as $t = 0$. We investigate the relaxation of the state generated by this process at sufficiently large times $t > 0$ when the current decay becomes logarithmic. In particular, we calculate the relaxation of the magnetic flux measured in two mutually perpendicular directions in the plane of the plate. In this analysis we imply that the barriers U_0 and U_1 are sufficiently large, $U_0/T \gg 1$, $U_1/T \gg 1$, in order to guarantee the existence of the logarithmic regime; the angle ζ exceeds $\mu \equiv k_c d/2$, the full rotation angle of the induction $\mathbf{B}$ in the sample (in other words, the fully penetrated critical state relative to the rotation of $\mathbf{B}$ is implied to occur in the sample). We also assume here that H considerably exceeds the variation of B across the thickness of the plate, $H \gg dj_{c\perp}/2$. The latter assumption enables us to use the approximation [9]: $j_{c\perp} = \text{const}$, $k_c = \text{const}$ (i.e. these quantities are independent of B), which is an extension of the well-known Bean critical state model, $j_{c\perp} = \text{const}$. In this approximation, one has [9] $j_{c\parallel} = k_c H$, and the above assumption $H \gg dj_{c\perp}/2$ can be rewritten as $\chi \gg \mu$ where $\chi \equiv j_{c\parallel}(H)/j_{c\perp}$.

We now present final formulas which enable one to extract μ (i.e., k_c) and U_1 from experimental data under the condition $\chi \gg \mu$. The general case when this condition is not fulfilled as well as details of the consideration will be given elsewhere [10]. We consider here the fluxes per unit length of the plate (the length is measured in the direction perpendicular to the flux). The flux along the final direction of the external field defined by the angle ζ, $\Phi_\parallel$, and the flux perpendicular to this direction, $\Phi_\perp$, are given by

$$\frac{\Phi_\parallel}{\Phi_{eq}} \approx \frac{\sin\mu}{\mu}\,, \qquad (2)$$

$$\frac{\Phi_\perp}{\Phi_{eq}} \approx \frac{1-\cos\mu}{\mu}\,, \qquad (3)$$

where $\Phi_{eq} = \mu_0 H d$ is the flux in the equilibrium state of the superconductor. Interestingly, the fluxes $\Phi_\parallel$, $\Phi_\perp$ are mainly determined by μ, while the parameter χ enters only corrections to these formulas. This enables one not only to find μ from appropriate experiments, but also to verify the fulfilment of the condition $\chi \gg \mu$ in these experiments. Note also that Eqs. (2) and (3) do not contain ζ; thus, the independence of experimental $\Phi_\parallel$ and $\Phi_\perp$ from this angle indicates that the fully penetrated critical state has been reached in the plate.

In what follows we shall always assume that $\Phi_\parallel > 0$, i.e., $0 < \mu < \mu_1$ where $\mu_1 \approx \pi$. When $\mu > \mu_1$, time-dependent instabilities are predicted to occur in the critical state of the plate [9] and for this reason we do not consider the case $\mu > \mu_1$ below. Note that the inequality $\mu < \mu_1$ leads to a restriction on the thickness of the plate from above.

The character of the magnetic relaxation turns out to be different in the intervals $0 \le \mu \le \mu_2$ and $\mu_2 < \mu < \mu_1$, where in our approximation ($\chi \gg \mu$), the boundary μ_2 separating these intervals satisfies the equation:

$$\cos\mu + \mu\sin\mu - 1 = 0\,, \qquad (4)$$

i.e., $\mu_2 \approx 0.742\pi$. When $0 \le \mu \le \mu_2$, the creep rates $\tilde{\Phi}_\parallel \equiv d\Phi_\parallel/d\ln t$ and $\tilde{\Phi}_\perp \equiv d\Phi_\perp/d\ln t$ are described by:

$$\frac{\tilde{\Phi}_\parallel}{\Phi_{eq}} \approx \frac{T}{U_1}\frac{\sin\mu - \mu\cos\mu}{\mu} + \frac{\tilde{\Phi}_{\rm Bean}}{\Phi_{eq}}\,, \qquad (5)$$

$$\frac{\tilde{\Phi}_\perp}{\Phi_{eq}} \approx \frac{T}{U_1}\frac{\cos\mu + \mu\sin\mu - 1}{\mu}\,, \qquad (6)$$

where the term, $\tilde{\Phi}_{\rm Bean} = (T/U_0)\mu_0 j_{c\perp} d^2/4$, corresponds to the creep rate in the usual Bean critical state [this expression for $\tilde{\Phi}_{\rm Bean}$ is equivalent to Eq. (1)]. If $U_1 \sim U_0$, this term is relatively small ($\sim \mu/\chi$) and may be omitted. But it becomes important at small μ, when the first term in Eq. (5) tends to zero as μ^2. At $\mu = \mu_2$, the rate $\tilde{\Phi}_\perp$ reaches zero, and when $\mu_2 < \mu < \mu_1$, the transverse flux $\Phi_\perp$ is locked in the sample, $\tilde{\Phi}_\perp = 0$, and its relaxation will occur in a later stage of the process. As to $\tilde{\Phi}_\parallel$ for $\mu_2 < \mu < \mu_1$, one has

$$\frac{\tilde{\Phi}_\parallel}{\Phi_{eq}} \approx \frac{T}{U_1}\frac{(2 - 2\cos\mu - \mu\sin\mu)}{\mu\sin\mu} + \frac{\tilde{\Phi}_{\rm Bean}}{\Phi_{eq}}\,. \qquad (7)$$

At $\mu \to \mu_1$, the rate $\tilde{\Phi}_\parallel$ diverges, which reflects the fact that the critical state becomes unstable at $\mu = \mu_1$.

We have shown that if $H \gg dj_{c\perp}/2$, formulas (2), (3), (5) - (7) enable one to extract from experimental data the longitudinal critical current density, $j_{c\parallel}$, and the effective height U_1 of the barrier that prevents flux-line cutting in the superconductor.

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