

# Properties of MgB<sub>2</sub> in a two-gap superconductivity model

T.Örd<sup>a,1</sup>, N.Kristoffel<sup>b</sup>, K.Rägo<sup>a</sup>

<sup>a</sup>*Institute of Theoretical Physics, University of Tartu, Tähe 4, 51010 Tartu, Estonia*  
<sup>b</sup>*Institute of Physics, University of Tartu, Riia 142, 51014 Tartu, Estonia*

---

## Abstract

For MgB<sub>2</sub> where coexist two coupled superconductivity gaps a two-band scheme has been developed. Three interaction channels have been taken into account: a pair-transfer type  $\sigma - \pi$ -interband repulsion, a  $\sigma$ -intraband effective attraction of electron-phonon nature, and a  $\sigma$ -intraband Coulomb interaction. The calculated temperature dependencies of gaps, heat capacity and  $H_{c2}$  agree with the experimental findings. The theoretical curve of  $T_c$  vs  $x$  for Mg<sub>1-x</sub>Al<sub>x</sub>B<sub>2</sub> follows the experimental data.

*Key words:*

MgB<sub>2</sub>; two-band model; superconductivity and thermodynamic characteristics

---

A number of experiments point to the two-gap nature of the MgB<sub>2</sub> superconductivity [1-6]. The electron structure calculations [7,8] also support this conclusion by revealing the Fermi level intersection by boron  $\sigma$ - and  $\pi$ -bands. At the same time, there is no doubt in the presence of strong  $\sigma$ -intraband pairing interaction in MgB<sub>2</sub> [7,8], however, the mentioned circumstances suggest to introduce the interband pairing channels and the use of two-band models of superconductivity [5,9-12].

The linearized Hamiltonian of the system incorporating electron-phonon and Coulomb interactions in the effective  $\sigma$ -band, and the  $\sigma - \pi$  scattering of intraband pairs is taken in the form

$$H = \sum_{\alpha\mathbf{k}s} \tilde{\epsilon}_\alpha(\mathbf{k}) a_{\alpha\mathbf{k}s}^+ a_{\alpha\mathbf{k}s} - \sum_{\alpha\mathbf{k}} \Delta_{\alpha\mathbf{k}} \langle a_{\alpha\mathbf{k}\uparrow}^+ a_{\alpha-\mathbf{k}\downarrow}^+ \rangle + \sum_{\alpha\mathbf{k}} (\Delta_{\alpha\mathbf{k}} a_{\alpha\mathbf{k}\uparrow}^+ a_{\alpha-\mathbf{k}\downarrow}^+ + \Delta_{\alpha\mathbf{k}}^* a_{\alpha-\mathbf{k}\downarrow} a_{\alpha\mathbf{k}\uparrow}), \quad (1)$$

where the superconductivity order parameters are defined as  $\Delta_{\alpha\mathbf{k}} = 2 \sum_{\beta\mathbf{k}'} W_{\alpha\beta}(\mathbf{k}, \mathbf{k}') \langle a_{\beta-\mathbf{k}'\downarrow} a_{\beta\mathbf{k}'\uparrow} \rangle$ . The band energies ( $\alpha = 1$  for  $\sigma$  and  $\alpha = 2$  for  $\pi$ ) read  $\epsilon_\alpha = \tilde{\epsilon}_\alpha + \mu$ , where  $\mu$  is the chemical potential.

<sup>1</sup> Corresponding author. E-mail: teetord@ut.ee

Other common designations are used. The gap equations ( $\Theta = k_B T$ )

$$\Delta_{\alpha\mathbf{k}} = - \sum_{\beta\mathbf{k}'} W_{\alpha\beta}(\mathbf{k}, \mathbf{k}') \Delta_{\beta\mathbf{k}'} \xi_{\beta\mathbf{k}'} \quad (2)$$

with  $\xi_{\alpha\mathbf{k}} = E_\alpha^{-1}(\mathbf{k}) \tanh[E_\alpha(\mathbf{k})/2\Theta]$  contain the quasi-particle energies  $E_\alpha(\mathbf{k}) = [\tilde{\epsilon}_\alpha^2(\mathbf{k}) + \Delta_{\alpha\mathbf{k}}^2]^{1/2}$ . The gaps are taken to be real.

Describing the s-wave superconductivity of MgB<sub>2</sub> the  $\sigma$ -intraband coupling constant  $W_{11} = V + U$  is supposed to contain a Coulombic part  $U > 0$  besides the electron-phonon attraction  $V < 0$  in the Debye-layer determined by  $\hbar\omega_D = 0.06$  eV [13]. The repulsive interband coupling is characterized by the constant  $W > 0$ . Interactions  $U$  and  $W$  are operative in the energy interval from  $E_c$  to zero ( $\sigma$ -band top). The cut-off energy  $E_c$  determines the bands overlap region and is taken as  $E_c = -2$  eV. Then the chemical potential of the undoped MgB<sub>2</sub> is  $\mu = -0.6$  eV [7]. We characterize the effective  $\sigma$ - and  $\pi$ -bands by constant densities of states  $\rho_1 = 0.25$  and  $\rho_2 = 0.11$  (eV<sup>-1</sup>) [7].

The necessary values of the interaction constants have been determined in [12] by simultaneous fitting of experimental data for  $T_c$ , the specific heat jump and the ratio of zero-temperature gaps. As a result  $V_1 =$

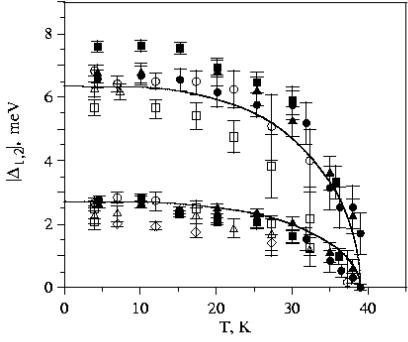


Fig. 1. The  $\text{MgB}_2$  superconductivity gaps *vs* temperature. Solid line - theory. Filled symbols - the experimental data of [2]; open symbols - the experimental data of [14].

$-1.01$  and  $W = 0.53$  (eV) have been chosen, and according to an estimation  $U = 1$  eV [12].

The temperature dependencies of the gaps on the Fermi level calculated from (2) agree well with the measured ones [2,14], as it is seen from Fig. 1.

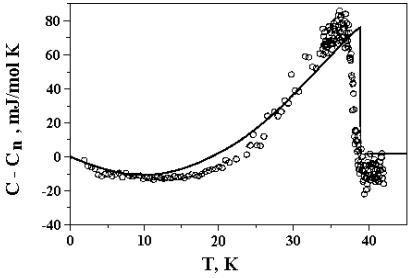


Fig. 2. The  $\text{MgB}_2$  specific heat *vs* temperature. Solid line - theory. Points represent the experimental data of [15].

On the basis of Eqs. (1),(2) one can find the thermodynamic characteristics for  $\text{MgB}_2$ . Theoretical curve of specific heat *vs* temperature in Fig.2 follows the experimental data of [15]. In Fig.3 the calculated temperature dependence of  $H_{C2}$  describes satisfactory the experimental data of [16]. At this the Ginzburg-Landau parameter value  $\kappa=38$  [4] has been used.

For the calculation of  $T_c(x)$  for  $\text{Mg}_{1-x}\text{Al}_x\text{B}_2$  we have taken account of  $\rho_{1,2}$  changes in the Debye layer near  $E_F$  with doping according to[8]. The result is shown in Fig.4 together with the experimental points from [17].

In conclusion, the model of present type seems to be able to describe the properties of the two-gap superconductor  $\text{MgB}_2$ .

## Acknowledgements

This work was supported by the Estonian Science Foundation Grant No 4961.

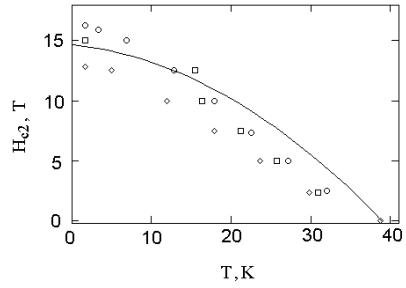


Fig. 3. The  $\text{MgB}_2$  critical magnetic field  $H_{C2}$  *vs* temperature. Solid line - theory. Points represent the experimental data of [16].

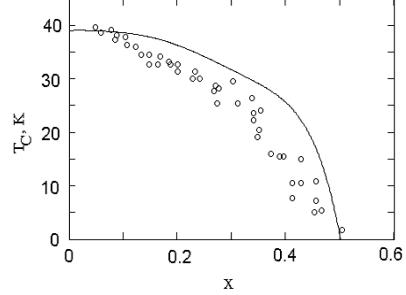


Fig. 4. The influence of doping on  $T_c$  in  $\text{Mg}_{1-x}\text{Al}_x\text{B}_2$ . Solid line - theory. Points represent the experimental data of [17].

## References

- [1] S.Tsuda et al., Phys. Rev. Lett. **87** (2001) 177006.
- [2] P.Szabo et al., Phys. Rev. Lett. **87** (2001) 137005.
- [3] X.K.Chen et al., Phys. Rev. Lett. **87** (2001) 157002.
- [4] Y.Wang, T.Plackowski, and A.Junod, Physica C **355** (2001) 179.
- [5] F.Bouquet et al., Phys. Rev. Lett. **87** (2001) 047001.
- [6] F.Giubileo et al., Phys. Rev. Lett. **87** (2001) 177008.
- [7] J.M.An and W.E.Pickett, Phys. Rev. Lett. **86** (2001) 4366.
- [8] J.Kortus et al., Phys. Rev. Lett. **86** (2001) 4656.
- [9] A.Y.Liu, I.I.Mazin, and J.Kortus, Phys. Rev. Lett. **87** (2001) 087005 .
- [10] A.A.Golubov et al., cond-mat/0111262 v 1 (2001).
- [11] E.Bascones and F.Guinea, Phys. Rev. B **64** (2001) 214508.
- [12] T.Örd and N.Kristoffel, Physica C **370** (2002) 17.
- [13] S.L.Bud'ko et al., Phys. Rev. Lett. **86** (2001) 1877.
- [14] I.Bugoslavsky et al., cond-mat/0110296 (2001).
- [15] R.K.Kremer, B.J.Gibson, and K.Ahn, cond-mat/0102432 v 2 (2001).
- [16] S.L.Bud'ko et al., Phys. Rev. B **65** (2001) 220503(R).
- [17] P.Pastorino et al., cond-mat/0106356 (2001).