Collective charge modes in a 1D electron liquid

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Abstract

A new behavior of the collective mode in a 1D electron liquid is found. The charge mode frequency goes to zero when the wave-number is close to double Fermi wave-number, i.e. the soft mode appears in addition to common long-wave plasmons. This mode is related to dynamic short-range electron correlations, which are adequately described in the frame of the Luttinger model. The soft mode is immanent in 1D and is absent in higher-dimensional systems. The results are valid for both Coulomb and short-range electron-electron interaction.

Key words: strongly correlated electrons, quantum wires, 1D plasmons, charge-density waves

1. Introduction

Electron-electron interaction is known to change dramatically the picture of elementary excitations in one-dimensional (1D) systems, giving rise to a strongly correlated quantum liquid [1]. The correlations in 1D electron liquid attract a great deal of interest, but a little is known about their manifestation in the collective mode spectrum, which is available directly from optical experiments [2].

The collective modes in 1D were thoroughly investigated within the random phase approximation with various modifications. In such a way no qualitatively new results were obtained as compared to higher dimensions, giving only the long-wave plasmon branch [3]. The flaw of this approach is obviously the incorrect account of the dynamic short-range correlations, inherent in 1D.

The short-range correlations naturally appear in the Luttinger liquid (LL) theory [4], which produces the $2k_{\rm F}$ component in the density response in addition to the usual long-wave one. This component (referred also to as the charge density wave, CDW) describes the spatial oscillation with a wave number close to double Fermi wave number $2k_{\rm F}$.

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In the present paper we show that a qualitatively new behavior of the collective charge mode appears owing to the CDW component of the electron density. The mode frequency goes to zero for the wave numbers close to $2k_{\rm F}$, i.e. a soft mode appears.

The spectrum of the collective charge mode can be found from the dynamic susceptibility $\chi(q,\omega)$. The latter has two components in 1D systems, since so does the electron density operator [5,6]:

$$\rho(x) = -\frac{1}{\pi}\partial_{\rm x}\phi + \frac{1}{2\pi}\partial_{\rm x}\sin(2k_{\rm F}x - 2\phi),$$

$\phi(x)$ being the bosonic phase. Here the first component $\rho_{\rm lw}$ describes long-wave excitations. The second component $\rho_{\rm CDW}$ describes the short-range electron correlations.

In order to find the collective mode spectrum, it is sufficient to analyze the singularities of the dynamic structure factor $S(q,\omega)$, which is proportional to the imaginary part of the susceptibility. The long-wave part of the structure factor $S_{\rm lw}(q,\omega)$ has the δ-singularity, corresponding to the excitation of one boson [1]:

$$S_{\rm lw}(q,\omega) = \left(1 - {\rm e}^{-\frac{\omega}{T}}\right)^{-1}|q|g(q)\delta(\omega-\omega_{\rm q}),$$

where $\omega_{\rm q} = |q|v_{\rm F}/g(q)$ is the boson dispersion law, $g(q)$ is the interaction parameter, and T is the temperature. Thus we find that the long-wave collective ex-

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citations (plasmons) are identical to Luttinger liquid bosons, and represent one-boson processes.

The dynamic electron correlations on the scale of the Fermi wave length are described by the CDW density response function $\chi_{\rm CDW}(x,t)$. For the short-ranged electron-electron interaction we find (see also [5]):

$$\chi_{\rm CDW}(x,t) = \Lambda\partial_{\rm x}^2\left[\cos(2k_{\rm F}x)|\sinh\xi|^{-g}|\sinh\zeta|^{-g}\theta(\xi\zeta)\right],$$

where $\xi,\zeta = (t \pm x/v)/T$, Λ is given in [7].

At zero temperature, the corresponding structure factor $S_{\rm CDW}(q,\omega)$ is non-zero inside the band $\omega_{|{\rm q}|-2{\rm k_F}} < \omega$, where many bosons are excited. At the band edges $S_{\rm CDW}(q,\omega)$ diverges as [8]

$$S_{\rm CDW}(q,\omega) \sim \frac{{\rm e}^{-4\beta|\ln\epsilon|^{1/2}}}{\epsilon|\ln\epsilon|^{1/2}}$$

for Coulomb e-e interaction, and as

$$S_{\rm CDW}(q,\omega) \sim \epsilon^{g-1},$$

for short-ranged interaction, where $\beta = [\pi\hbar v_{\rm F}/2{\rm e}^2]^{1/2}$, $\epsilon = \omega - \omega_{|{\rm q}|-2{\rm k_F}} \to +0$.

The singularities of $S_{\rm CDW}(q,\omega)$ show that the new collective mode appears near to $q = 2k_{\rm F}$. The mode dispersion is $\omega = \omega_{|{\rm q}|-2{\rm k_F}}$. Here ω is pure real, which means that at $T = 0$ the $2k_{\rm F}$ mode is non-decaying. An important conclusion is that the mode frequency goes to zero at $q \to 2k_{\rm F}$, in other words, the mode is soft. It is the presence of the $2k_{\rm F}$ mode that principally distinguishes the collective excitations in the Luttinger liquid from those obtained within other approaches that do not take adequately into account the dynamic electron correlations of the short-range scale. The dispersion of collective modes in 1D is illustrated by Fig. 1.

At finite temperature $S_{\rm CDW}(q,\omega)$ can be exactly calculated for the short-ranged interaction. Denote $p = |q| - 2k_{\rm F}$, $\omega_\pm = (\omega \pm pv)/2T$ to express $S_{\rm CDW}(q,\omega)$ in the form $S_{\rm CDW}(q,\omega) = vk_{\rm F}^2 S(\omega_+)S(\omega_-)$, where

$$S(\omega) = C(\omega)\left[\cos(\pi g) - \cosh(\pi\omega)\right]^{-1},$$

$C(\omega)$ being the nonsingular function of ω [9]. $S_{\rm CDW}(q,\omega)$ has a simple pole at $\omega_\pm = -{\rm i}g$. Thus the mode dispersion is $\omega = \omega_{|{\rm q}|-2{\rm k_F}} - 2{\rm i}gT$. This means that the mode gets the decrement $\gamma = 2gT$, which is related to Landau damping. The decrement is independent of the wave-number q, so at $q \to 2k_{\rm F}$ it becomes comparable to the mode frequency, which goes to zero. We see that the thermal fluctuations destroy the modulated density distribution, as it should be in one dimension, and the ground state remains uniform. Nonetheless, as one moves from $q = 2k_{\rm F}$, the mode becomes well-defined. Note that for sufficiently strong electron-electron interaction, e.g. for Coulomb interaction when $g \to 0$, the region, where the mode does not exist, is squeezed to one point $q = 2k_{\rm F}$, $\omega = 0$. For $\omega \neq 0$ the collective mode propagates freely.

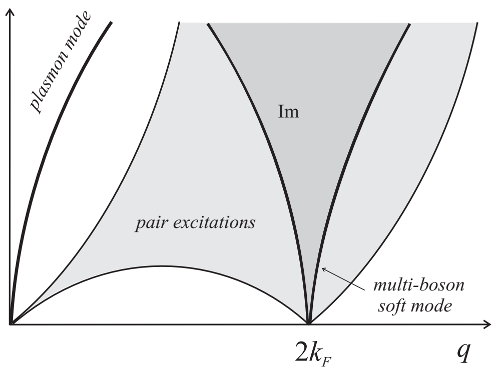

Fig. 1. The dispersion of collective charge modes in a 1D electron liquid. In the long-wave region there exist plasmons. In addition, there is a soft mode in the region close to $2k_{\rm F}$. This mode is related to dynamic short-range electron correlations. The dispersion line of the $2k_{\rm F}$ mode is the boundary of the multi-boson-excitation band (darkened in the figure), where the dissipation takes place. The mode itself is non-decaying at zero temperature.

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[7] $\Lambda = \frac{1}{2\pi h}\left(\frac{\pi g T}{\epsilon_{\rm F}}\right)^{2g}\sin(\pi g)$

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[9] $C(\omega) = \frac{2^{\frac{g-5}{2}}}{T}\left(\frac{\pi g T}{\epsilon_{\rm F}}\right)^g {\rm e}^{\frac{\pi\omega}{2}}\,\Gamma^{-1}(g)\left|\Gamma(1-\frac{g}{2}-\frac{{\rm i}\omega}{2})\right|^{-2}$