Inductive coupling of two superconducting loops with three Josephson junctions

Yoshihiro Shimazu [1]

Department of Physics, Yokohama National University, Yokohama 240-8501, Japan

Abstract

A superconducting loop with three Josephson junctions can behave as a macroscopic quantum two-level system, which could be used as a flux qubit. Coupling of qubits is necessary to realize a quantum computer. We studied the classical states of two such loops which are inductively coupled. When the magnetic flux in each loop is near half a flux quantum, there are two classical states in which the magnetic moments of the loops are parallel and anti-parallel respectively while the anti-parallel state is energetically favorable. The magnetic flux due to the coupled loops is calculated as a function of the frustration.

Key words: Josephson junction ;qubit ;mutual inductance

1. Introduction

The experimental demonstration of the quantum coherent oscillations in Josephson junction circuits has suggested the possibility that solid-state quantum information processors can be made of Josephson-junction qubits.[1,2] Several Josephson-junction qubit schemes have been proposed and investigated experimentally. A superconducting loop with three Josephson junctions can be used as a flux qubit which has two classical persistent-current states of opposite polarity.[3,4] Microwave-spectroscopy on a single three-junction loop indicated a finite tunnel splitting which is a clear evidence for quantum superposition of the persistent-current states.[5] A controlled operation on one qubit and that on two qubits are necessary to realize a quantum computer. The latter operation requires an inter-qubit coupling. We investigated the behavior of two qubits (three-junction loops) which are inductively coupled. This system can be regarded as coupled spin-1/2 particles. The analysis in the present paper is restricted to the classical regime in which the quantum coherence of the qubit is neglected. In the classical regime, this system could be used to build a novel logic gate of a classical Josephson computer.

2. Model of coupled qubits

The total Josephson energy U of a three-junction loop of which two junctions have Josephson energies E_J and the third one has a Josephson energy α times larger is

$$U/E_\mathrm{J} = 2 + \alpha - \cos\gamma_1 - \cos\gamma_2 - \alpha\cos(2\pi f + \gamma_1 - \gamma_2), \quad (1)$$

where γ_1 and γ_2 are the gauge-invariant phases of the junctions, and f is the magnetic frustration in the loop.[3,4] The stable classical states correspond to energy minima in $U(\gamma_1, \gamma_2)$. The circulating current in the classical state is given by $I = I_\mathrm{c}\sin\gamma_1$. Near $f = 1/2$ the system has two classical states of opposite polarity of the circulating currents. The thermally averaged value of the circulating current $I_\mathrm{Q}(f)$ is then determined as a function of the frustration in the loop.

[1] E-mail: yshimazu@ynu.ac.jp

We consider a model where two identical qubits with the self inductances L are coupled through the mutual inductance M. The effective frustrations in the qubits f_1 and f_2 are related to the applied frustration $f_{\rm ext}$ and the circulating currents of the qubits I_1 and I_2 by the equations,

$$f_1 = f_{\rm ext} + LI_1/\Phi_0 + MI_2/\Phi_0 \tag{2}$$

$$f_2 = f_{\rm ext} + LI_2/\Phi_0 + MI_1/\Phi_0, \tag{3}$$

where $I_1 = I_{\rm Q}(f_1)$, $I_2 = I_{\rm Q}(f_2)$ and Φ_0 is the flux quantum. Solving these couple equations, we obtained stable classical states of the coupled qubits as a function of the applied frustration $f_{\rm ext}$.

3. Results

The coupled system has two stable classical states in the vicinity of $f_{\rm ext} = 1/2$. Figures 1 show the sum of the currents $I_1 + I_2$ as a function of $f_{\rm ext}$ in these states. The parameters of the system and the temperature are chosen to be $\alpha = 0.8$, $LI_{\rm c}/\Phi_0 = 0.04$, $MI_c/\Phi_0 = -0.02$ and $k_{\rm B}T/E_{\rm J} = 0.1$, where $E_{\rm J} = \frac{\Phi_0 I_{\rm c}}{2\pi}$. We note that the magnetic flux due to the qubits is proportional to $I_1 + I_2$, assuming that it is measured with a DC-SQUID which is equally coupled to the two qubits.

In the middle horizontal portion shown in Fig. 1(a), the currents in the qubits flow in the opposite direction, that is, the magnetic moments of the qubits are anti-parallel. This anti-parallel state suddenly disappears as $|f_{\rm ext} - 1/2|$ increases. Figure 1(b) shows the other stable state of the system. In this state the magnetic moments of the qubits are always parallel and of the same magnitude. At $f_{\rm ext} = 1/2$ the circulating currents are zero. We note that the step width of the magnetic flux is smaller than that of an uncoupled single qubit due to the effect of the neighboring qubit.

The total energy of the system is

$$E_{\rm tot} = U_1 + U_2 + \frac{1}{2}L(I_1^2 + I_2^2) + MI_1I_2, \tag{4}$$

where U_1 and U_2 are the Josephson energies of the qubits. The energies $E_{\rm tot}$ of the two states are shown in Fig. 2. The comparison shows that the anti-parallel state is energetically favored near $f_{\rm ext} = 1/2$ although the terms of the magnetic energy is lower in the parallel state. This result indicates that the inductive coupling brings about the anti-parallel state in reality.

We expect these results will be verified experimentally.

References

[1] Y. Nakamura *et al*, Nature **398** (1999) 786.

[2] D. Vion *et al*, Science **296** (2002) 886.

[3] J.E. Mooij *et al*, Science **285** (1999) 1036.

[4] T.P. Orlando *et al*, Phys. Rev. B**60** (1999) 15398.

[5] Caspar H. van der Wal *et al*, Science **290** (2000) 773.

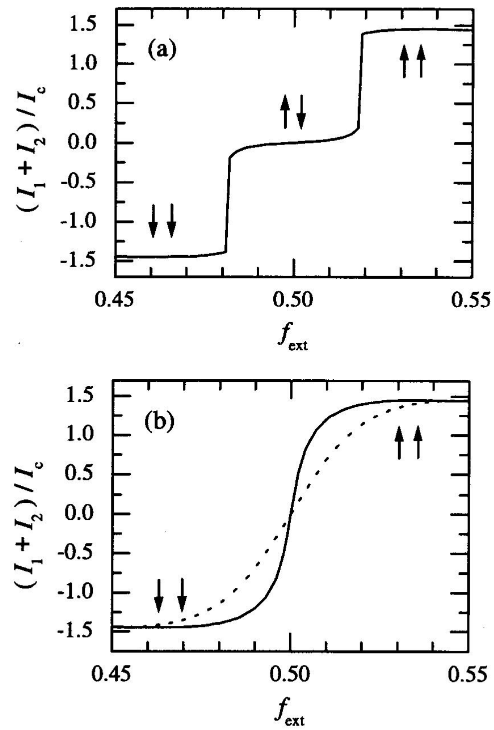

Fig. 1. The sum of the circulating currents of the two qubits as a function of the applied frustration $f_{\rm ext}$ for the two classical states. The magnetic moments are (a) anti-parallel and (b) parallel, respectively, near $f_{\rm ext} = 1/2$ in these states. The arrows show the orientation of the magnetic moments of the two qubits. The dotted line in (b) shows the twice circulating current of an uncoupled single qubit.

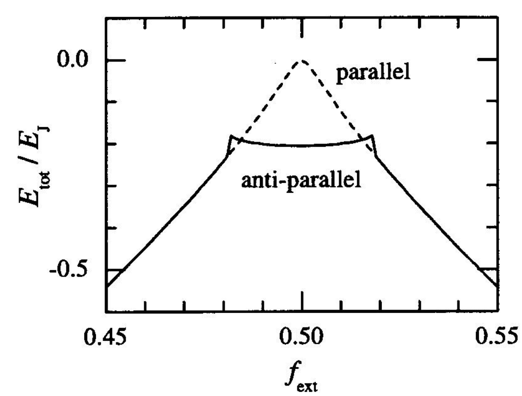

Fig. 2. The total energies of the coupled qubits in the two states in which the magnetic moments are paralle and anti-parallel.