One-particle model of the metal-insulator transition at $T = 0$

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Abstract

A metal-insulator transition (MIT) in two-dimensional systems at $T = 0$ is regarded as a consequence of flattening the (in fact three-dimensional) electronic waveguide which is formed in heterostructures due to application of a confinement potential. The waveguide flattening is accompanied by a stick-slip change in the number of extended waveguide modes (conducting channels), of which the last mode disappearance is identified as MIT.

Key words: Metal-insulator transition; disordered systems; electron localization; quantum dephasing

1. Introduction

According to the scaling theory of localization [1], there can be no metallic ground state of two-dimensional (2D) systems at any strength of disorder. This assertion, however, is in apparent conflict with a bulk of experimental results obtained in recent decades on different 2D electron and hole systems, which clearly show the resistance anomalies characteristic for metal-insulator transition.

Although many ideas were put forward to resolve the inconsistency between experimental findings and theoretical predictions (see extensive discussion in [2]), the subject still remains quite problematic. In [3], it was proven that the metallic state should not be considered as unusual for 2D disordered systems. At the same time, a clear scenario capable of explaining MIT observed in two dimensions was not proposed.

In this paper, the method of [3] previously developed for exactly two-dimensional systems is extended to realistic 3D systems of waveguide type in order to take into account the finite extent of electron wave functions in the direction normal to the interface. By monitoring the width of the near-surface potential well which shapes the electronic waveguide one can change the number of conducting channels and, consequently, two-dimensional density of current carriers. The last channel closing is identified with a MIT, which is thus recognized as a true quantum phase transition.

2. Reduction to one-dimensional problem

By simulating the near-surface potential well in the form of a flattened electronic waveguide, the equation for one-particle retarded 3D propagator $G(\mathbf{r}, \mathbf{r}')$ can be reduced to an infinite set of one-coordinate differential equations for its mode Fourier-components $G_{\mu\mu'}(x, x')$. In the case of a rectangular waveguide cross-section the equations read

$$\left[\frac{\partial^2}{\partial x^2} + \kappa_\mu^2 + \mathrm{i}0 - U_{\mu\mu}(x)\right] G_{\mu\mu'}(x, x') - \sum_{\nu\neq\mu} U_{\mu\nu}(x) G_{\nu\mu'}(x, x') = \delta_{\mu\mu'}\delta(x - x') \ . \quad (1)$$

Here, $\boldsymbol{\mu} = (n, m)$ is the two-component vectorial mode index ($n, m \in \aleph$), x is the lengthwise coordinate in the waveguide, $x \in (-L/2, L/2)$, $U_{\mu\nu}(x)$ are the mode matrix elements of the random potential $V(\mathbf{r})$,

$$U_{\mu\nu}(x) = \int_S \mathrm{d}\mathbf{r}_\perp |\mathbf{r}_\perp; \boldsymbol{\mu}\rangle V(\mathbf{r}) \langle \mathbf{r}_\perp; \boldsymbol{\nu}| \ , \quad (2)$$

the parameter

$$\kappa_\mu^2 = k_F^2 - (\pi n/W)^2 - (\pi m/H)^2 \ , \quad (3)$$

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with k_F being the Fermi wavenumber, has the meaning of a longitudinal energy of the mode $\boldsymbol{\mu}$. In Eq. (2), integration is carried out within the bounds of the waveguide cross-section area $S = WH$, W and H are its width and height, respectively. The potential $V(\mathbf{r})$ is specified by the zero mean value, $\langle V(\mathbf{r})\rangle = 0$, and the binary correlation function $\langle V(\mathbf{r})V(\mathbf{r}')\rangle = \mathcal{Q}\mathcal{W}(x - x')\delta(\mathbf{r}_\perp - \mathbf{r}'_\perp)$.

From (1), all the *inter-mode* propagators $G_{\nu\mu}(x, x')$, with $\boldsymbol{\nu} \neq \boldsymbol{\mu}$, can be expressed, by means of a linear operation, in terms of the corresponding intra-mode propagators $G_{\mu\mu}(x, x')$. This allows one to derive the exact one-dimensional closed equations for the diagonal mode Green functions only,

$$\left[\frac{\partial^2}{\partial x^2} + \kappa_\mu^2 + \mathrm{i}0 - U_{\mu\mu}(x) - \hat{\mathcal{T}}_\mu\right] G_{\mu\mu}(x, x') = \delta(x - x') \,. \quad (4)$$

Here

$$\hat{\mathcal{T}}_\mu = \boldsymbol{P}_\mu \hat{\mathcal{U}} (\mathbf{1} - \hat{\mathsf{R}})^{-1} \boldsymbol{P}_\mu \,, \quad (5)$$

is the effective non-local potential (T-matrix) which exactly takes into account all inter-mode scattering events. The operators $\hat{\mathcal{U}}$ and $\hat{\mathsf{R}}$ are specified in mixed coordinate-mode space $(x, \boldsymbol{\mu})$ by matrix elements $|x, \boldsymbol{\mu}\rangle\hat{\mathcal{U}}\langle x', \boldsymbol{\nu}| = U_{\mu\nu}(x)\delta(x - x')$ and $|x, \boldsymbol{\mu}\rangle\hat{\mathsf{R}}\langle x', \boldsymbol{\nu}| = G_\nu^{(V)}(x, x')U_{\nu\mu}(x')$, $\boldsymbol{P}_\mu$ is the projection operator into the mode $\boldsymbol{\mu}$. The function $G_\nu^{(V)}(x, x')$ is the solution of equation (1) with only intra-mode potentials kept.

The equation (4) spectrum can be effectively analyzed in the weak scattering limit corresponding to the inequalities k_F, $r_c \ll \ell$, where r_c is the potential $V(\mathbf{r})$ correlation radius, ℓ is the electron mean free path. The inter-mode scattering accounted for by T-matrix (5) leads to the mode state $\boldsymbol{\mu}$ dephasing, whose rate equals

$$\frac{1}{\tau_\mu^{(\varphi)}} = \frac{\mathcal{Q}}{4S}{\sum_{\nu\neq\mu}}' \frac{1}{\kappa_\nu}\left[\widetilde{\mathcal{W}}(\kappa_\mu - \kappa_\nu) + \widetilde{\mathcal{W}}(\kappa_\mu + \kappa_\nu)\right] \,. \quad (6)$$

The prime at the sum index indicates the summation over open channels only, $\widetilde{\mathcal{W}}(q)$ is the Fourier transform of $\mathcal{W}(x)$. It follows from Eq. (6) that quenched disorder can cause the dephasing of the electron states properly classified subject to the system boundedness, but this is the case exclusively in the conductors with more than one extended mode.

3. The conductance

With the solution of Eq. (4), the average conductance obtained from the linear response theory reads

$$\langle g(L)\rangle = {\sum_\mu}' \frac{l_\mu^{(\varphi)}}{L}\left[1 - \frac{l_\mu^{(\varphi)}}{L}\exp\left(-\frac{L}{l_\mu^{(\varphi)}}\right)\sinh\frac{L}{l_\mu^{(\varphi)}}\right] \,. \quad (7)$$

Here $l_\mu^{(\varphi)} = 2\kappa_\mu\tau_\mu^{(\varphi)}$ is the mode $\boldsymbol{\mu}$ dephasing length. The conductance (7) depends crucially on the number of open channels. It can be seen from Eq. (3) that this number can be varied by changing either of waveguide transverse dimensions, e.g., the thickness H. By strengthening the applied depletion voltage, the number of extended modes can be gradually reduced, so that only evanescent modes, with $\kappa_\mu^2 < 0$, remain eventually in the conductor. These modes are strongly localized at length scales comparable with de Broglie wavelength k_F^{-1}, so that in this case the electron system behaves like an insulator.

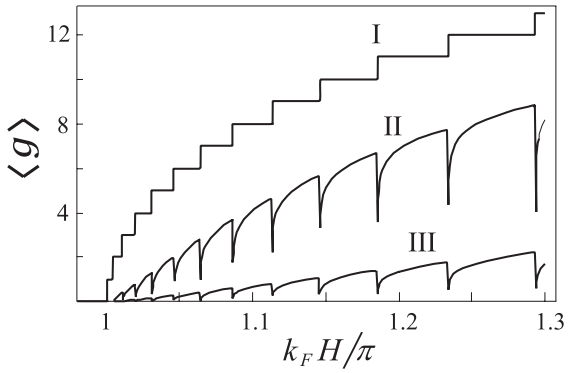

Fig. 1. The dimensionless conductance dependence on the waveguide thickness H at a fixed value of width ($k_F W/\pi = 20.5$). The curve I corresponds to $\lambda = L/\ell = 0$, II — $\lambda = 0,5$, III — $\lambda = 5$.

In Fig. 1, the dependence of the conductance (7) on the parameter $k_F H$ is depicted for different disorder strengths. In the ballistic limit, $L/\ell \to 0$, the conductance exhibits clear stepwise behaviour with the jump value exactly equal to the conductance quantum $e^2/\pi\hbar$. As the degree of disorder grows, the height of the steps is lowered, their shape is smoothed out, and the conductance approaches the well-known Drude value.

The transition of a planar (quasi-2D) electron system to its 0-mode "dielectric" state, which corresponds to the leftmost forepart of the curves in Fig. 1, takes place almost independently of the degree of disorder. This implies that the above described MIT is caused not by the Anderson localization of carrier states or by Coulomb interaction of carriers, but rather it is a true quantum phase transition related to size quantization.

References

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