

# Scattering of atoms from liquid helium films and slabs

E. Krotscheck<sup>1</sup>, V. Apaja and A. Rimmnag

*Institut für Theoretische Physik, Johannes-Kepler Universität, A 4040 Linz, Austria*

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## Abstract

We report new results in describing the scattering of  $^4\text{He}$  atoms off  $^4\text{He}$  films and slabs. The description incorporates elastic and inelastic scattering off the host liquid. We have computed the probabilities of elastic and inelastic reflection, sticking and, in the case of slabs, transmission for normal incidence.

*Key words:* Quantum fluids; reflectivity; variational theory

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## 1. Introduction

The dynamics of liquid  $^4\text{He}$  with a free surface has recently been studied both experimentally[1] and theoretically[2]. An impinging atom excites surface modes with very high probability, which makes atom scattering an ideal probe for ripplon and third sound excitations. Apart from the much-studied helium films on substrates such as graphite or cesium new experimental results on scattering from helium slabs are expected to emerge soon.

Scattering of  $^4\text{He}$  atoms off liquid  $^4\text{He}$  films and slabs is generically a non-local problem with a completely symmetric many-body wave function of the total system. The impinging atoms cannot be simply treated as impurities, since they can lose energy to excitations in the host liquid and may become part of it. Therefore, scattering depends crucially on both the static and the dynamical structure of the liquid.

The present theory has been described in detail in Refs. [3] (static structure) and [4] (dynamical properties). The application to atom reflectivity is based on solving the effective Schrödinger equation[2]

$$H_1\Psi(\mathbf{r}) - \int d\mathbf{r}' \Sigma(\mathbf{r}, \mathbf{r}', \omega)\Psi(\mathbf{r}') \quad (1)$$

$$= \hbar\omega \int d\mathbf{r}' S(\mathbf{r}, \mathbf{r}')\Psi(\mathbf{r}') ,$$

where  $\rho(\mathbf{r})$  is the density profile of the host liquid and  $H_1$  stands for

$$H_1 = \frac{\hbar^2}{2m} \left[ -\nabla^2 + \frac{\nabla^2 \sqrt{\rho(\mathbf{r})}}{\sqrt{\rho(\mathbf{r})}} \right] . \quad (2)$$

The static structure factor  $S(\mathbf{r}, \mathbf{r}')$  serves for the symmetrization of the total system's wave function and is replaced by  $\delta(\mathbf{r}, \mathbf{r}')$  if host liquid and probe particles are not identical. The *complex* self energy  $\Sigma(\mathbf{r}, \mathbf{r}', \omega)$  accounts for the interaction between single-particle excitations in the host liquid (see Ref. [4]). In the present model an excitation may decay into two excitations of lower energy via three-phonon processes and thus the scattering atom may lose energy and get stuck in the film or slab. The liquid is assumed to be translationally invariant in the  $x-y$  plane and Eq. (1) is solved subject to the boundary conditions

$$\Psi(z) \rightarrow \begin{cases} e^{ik_{\perp}z} + Re^{-ik_{\perp}z} & \text{for } z \rightarrow -\infty \\ Te^{ik_{\perp}z} & \text{for } z \rightarrow \infty \end{cases} , \quad (3)$$

for slabs; for films only the  $z < 0$  side is used. Since the momentum component parallel to the liquid surface is zero, the perpendicular wave vector,  $k_{\perp}$ , is determined solely by the energy  $\omega$  of the incident atom.

If there is no energy loss, i.e., the single particle excitations in the host liquid are considered as noninter-

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<sup>1</sup> E-mail: Eckhard.Krotscheck@jku.at

acting quasiparticles, reflectivity and transmittivity of the incident particle add to unity,  $|R|^2 + |T|^2 = 1$ : In this case the single particle flux is conserved. The corresponding scattering states  $\Psi(\mathbf{r})$  are Feynman states with discrete bound states and a continuum of scattering states, delimited by the energy  $\hbar\omega = -\mu + \hbar^2 k_{\parallel}^2 / (2m)$ , where the chemical potential is  $\mu \approx -7$  K.

The self energy  $\Sigma(\mathbf{r}, \mathbf{r}', \omega)$  describes the interaction between single excitations in the host liquid and dresses the corresponding Feynman states. The real part of the self energy renormalizes the excitation energies and the imaginary part describes their finite lifetime. The fraction of inelastic scattering processes, i.e., sticking, inelastic reflectivity or transmittivity can be computed from the self energy by keeping track of the decay channels of states excited by the impinging atom.

## 2. Results

Figures 1 to 3 show the probabilities of elastic and inelastic scattering processes as a function of the incident energy  $\hbar\omega$  in the case of scattering from a helium film on cesium and from a freely floating slab. The film thickness is about 25 Å (six layers) and the slab thickness is about 80 Å. Finite thickness in the  $z$  direction gives rise to oscillations in the probabilities.

The results show quantum reflection at very low energies, at slightly higher energies sticking is the dominant process due to low-energy surface modes, which are now ripplons. As Fig. 2 shows, the low-energy reflectivity can drop to almost zero at some film coverages. Experimentally, we expect the higher-energy features to show up at slightly lower energies, because the excitation energies in the roton region are overestimated by the theory. In the case of films, the peaky structure near 14 K is due to layer rotons, and more structure is seen in stronger substrate potentials, where layering of helium atoms is more pronounced. Similar structure can be seen in the case of slabs, where it is due to excitation of ordinary rotons.

## Acknowledgements

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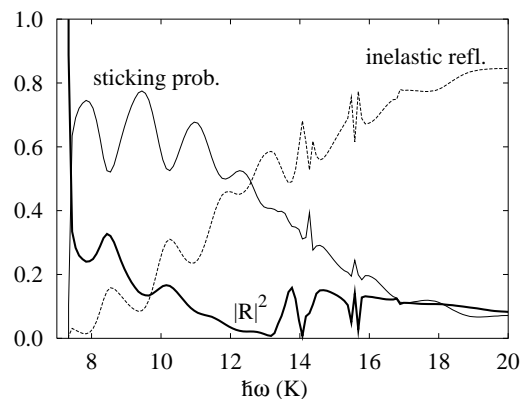


Fig. 1. Reflectivity  $|R|^2$  (thick line), sticking probability and inelastic reflectivity as a function of incident energy  $\hbar\omega$  of  $^4\text{He}$  atoms scattering from a thick helium film on cesium.

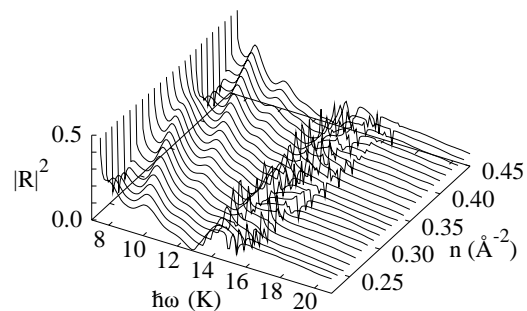


Fig. 2. Reflectivity from a thick helium film on cesium as a function of incident energy  $\hbar\omega$  for a set of coverages  $n$ .

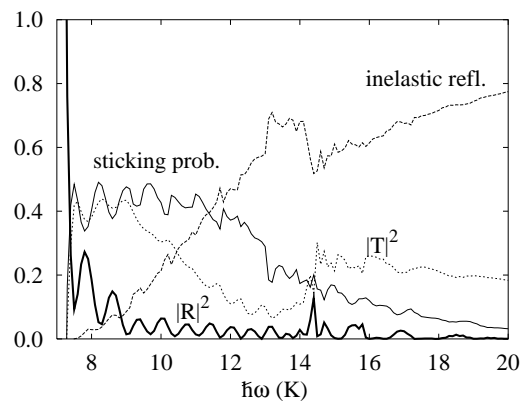


Fig. 3. Same as Fig. 1 in the case of a slab, with additional possibility for transmission. The probabilities add to unity.