Magnetization and dimerization profiles of open spin ladders

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Abstract

The physical properties of the edge states of the open two-leg generalized spin ladder are investigated by means of the bosonization approach. Depending on interchain couplings, two different types of Haldane gapped phases are obtained, one with spin 1/2 edge states, the other one without edge states. We determine the magnetization and dimerization profiles for both phases. Further applications to the spin-1 chain are discussed. Finally, we address the case of interchain interactions breaking SU(2) spin symmetry.

Key words: spin ladder; Haldane gap; impurities ; edge states

Using a non-linear sigma model with topological term description, Haldane showed in 1983 that spin chains with integer spin have gapped excitations above their ground state, whereas spin chains with half-odd integer spins would remain gapless[1]. Using the same non-linear sigma model formulation, it was later conjectured that a semi-infinite spin S chain has spin S/2 (respectively $S/2-1/4$) excitations near the edge for integer spins (respectively half-odd integer) S [2]. A simple explanation of the existence of free spin-1/2 moments at the ends a broken spin-1 chain can be obtained from the valence bond solid model[3] which provides an intuitive description of the ground state of the spin-1 chain. These spin-1/2 edge states have been observed in the Haldane-gap spin-1 compound NENP doped with nonmagnetic and magnetic impurities[4].

There exists a strong analogy between spin ladders and spin-1 chains[5]. In the present paper, we discuss edge states of the semi-infinite two-leg spin-1/2 ladder and the semi-infinite spin-1 chain. The Hamiltonian of the semi-infinite generalized two leg ladder[6] that we shall consider reads as follows:

$$H = J_\parallel \sum_{\substack{i=1,\infty \\ p=1,2}} \mathbf{S}_{i,p}\cdot\mathbf{S}_{i+1,p} + \sum_{i=1,\infty} J_\perp \mathbf{S}_{i,1}\cdot\mathbf{S}_{i,2} + J'_\perp \mathbf{S}_{i,1}\cdot\mathbf{S}_{i+1,2} + J''_\perp \mathbf{S}_{i,2}\cdot\mathbf{S}_{i+1,1}. \quad (1)$$

In a previous publication[7], we have discussed the fermionization of the above model (1) with $J'_\perp = J''_\perp = 0$. Using a similar approach, keeping only the most relevant terms, the continuum description of Hamiltonian (1) is given by $\mathcal{H} = \mathcal{H}_t + \mathcal{H}_s$ with:

$$\mathcal{H}_t = -\frac{iv}{2}\int_0^\infty dx \sum_{a=1}^3 (\xi_R^a \partial_x \xi_R^a - \xi_L^a \partial_x \xi_L^a) - i m_t \int_0^\infty dx \sum_{a=1}^3 \xi_R^a \xi_L^a,$$

$$\mathcal{H}_s = -\frac{iv}{2}\int_0^\infty dx \left(\xi_R^0 \partial_x \xi_R^0 - \xi_L^0 \partial_x \xi_L^0\right) - i m_s \int_0^\infty dx\, \xi_R^0 \xi_L^0, \quad (2)$$

where $v = \pi J_\parallel a/2$, $m_t = -(J_\perp - J'_\perp - J''_\perp)/(2\pi)$, $m_s =$

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$3(J_\perp - J'_\perp - J''_\perp)/(2\pi)$, with the boundary conditions $\xi^a_R(0) = \xi^a_L(0), a = 0, \dots, 3$. For $J'_\perp + J''_\perp > J_\perp$, spin-1/2 edge states are present at the extremity of the chain as a result of the boundary conditions. Using a mapping on non-critical Ising models[7], we have found the following staggered magnetization profiles induced by the presence of the edge state:

$$(-)^{x/a}\langle (S_1+S_2)^a(x)\rangle \sim e^{-m_t x/v}(x \gg v/m_t),$$
$$(-)^{x/a}\langle (S_1+S_2)^a(x)\rangle \sim x^{1/2}(x \ll v/m_t). \quad (3)$$

For $J_\perp > J'_\perp + J''_\perp$, edge states are absent, and there is no induced staggered magnetization. This is another indication that depending on the sign of $J_\perp - (J'_\perp + J''_\perp)$ one has two different types of spin liquid ground state[6]. A schematic view of these spin liquid ground state is shown on figure 1 for $J_\perp, J'_\perp \gg J_\parallel$.

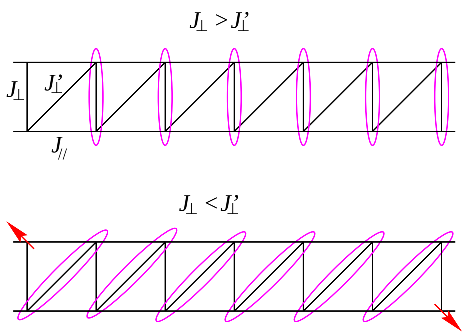

Fig. 1. The two different spin liquid states in the generalized spin ladder for $J''_\perp = 0$. In the first type of spin liquid state($J_\perp > J'_\perp$) singlets are vertical and edge states are absent, but in the second type of spin liquid singlets are diagonal and edge states are present.

For $J_\perp = J'_\perp + J''_\perp$ one must take into account subleading marginally relevant terms. The bosonized Hamiltonian is then similar to the one of the open two-leg zig-zag ladder defined by $J_\perp = J'_\perp$ and $J''_\perp = 0$[8]. There are strong indications [9] that in such cases edge states are absent. The case of the spin-1 chain is obtained from the spin ladder [10,11] by making $m_s = -\infty$, $m_t > 0$. Edge states are present, and the magnetization profile is then:

$$(-)^{x/a}\langle S^a(x)\rangle \sim e^{-m_t x/v}(x \gg v/m_t),$$
$$(-)^{x/a}\langle S^a(x)\rangle \sim x^{5/8}(x \ll v/m_t). \quad (4)$$

A staggered dimerization is also induced by the presence of the boundary. We obtain in the case of the ladder for $J'_\perp + J''_\perp > J_\perp$ and the spin-1 chain a staggered dimerization profile:

$$(-)^i \left\langle \sum_p \mathbf{S}_{i,p}\cdot\mathbf{S}_{i+1,p}\right\rangle \sim e^{-3m_t ia/v}(ia \gg v/m_t),$$

$$(-)^i \left\langle \sum_p \mathbf{S}_{i,p}\cdot\mathbf{S}_{i+1,p}\right\rangle \sim i^{-\beta}(ia \ll v/m_t), \quad (5)$$

with $\beta = 1/2$ for the ladder and $\beta = 3/8$ for the spin-1 chain. For the ladder with $J'_\perp + J''_\perp < J_\perp$, we obtain the same profile, with $3m_t \to m_s$. Now, we turn to the case of an anisotropic ladder or spin-1 chain. The Hamiltonian $\mathcal{H}_t$ reads:

$$\mathcal{H}_t = -\frac{iv}{2}\int_0^\infty dx \sum_{a=1}^{3} (\xi^a_R\partial_x\xi^a_R - \xi^a_L\partial_x\xi^a_L)$$
$$-\, i\int_0^\infty dx \sum_{a=1}^{3} m_a\, \xi^a_R\xi^a_L. \quad (6)$$

For $m_a > 0$, spin 1/2 edge states are present. However, they now have effective g factors such that $2g_c^{-1} = \sqrt{m_a/m_b} + \sqrt{m_b/m_a}$ with $a, b \neq c$. For the spin-1 chain, the staggered magnetization is now:

$$(-)^{x/a}\langle S^a(x)\rangle \sim e^{-m_a x/v} \quad (7)$$

and the staggered dimerization decays as $e^{-(m_1+m_2+m_3)x/a}$ for long distances.

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