Goldstone mode kink-solitons in double layer quantum Hall systems

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Abstract

It is shown that in charge unbalanced double layer quantum Hall system with zero tunneling pseudospin Goldstone mode forms moving kink-soliton in weakly nonlinear limit. This charge-density localization moves with a velocity of gapless linear spin-wave mode and could be easily observed experimentally. We predict that mentioned Goldstone mode kink-solitons define diffusionless charge transport properties in double layer systems, where mentioned kink-solitons could be considered as transport carriers.

Key words: Quantum Hall (Pseudo)ferromagnets;

The observation of split-off peaks of tunneling conductance versus in-plane magnetic field in recent experiments [1] in double layer quantum Hall systems with total Landau level filling factor $\nu = 1$ leads to obvious interpretation of these peaks as "footsteps" of linearly dispersing pseudospin Goldstone mode [2]. Thus the correctness and efficiency of field-theoretical approach according which the quantum Hall double layer system could be described as easy-plane type (pseudo)ferromagnet [3] is proved. The present paper is devoted to investigation of dynamical nonlinear properties of the mentioned Goldstone mode and, particularly, the existence of moving localized solutions.

The Goldstone mode naturally appears in systems with continuous symmetries as a result of spontaneous symmetry breaking which in presence of nonlinearity leads to the appearance of Goldstone mode kink-solitons [4]. The difference between the Goldstone mode kink-solitons and sine-Gordon kinks appearing in double layer quantum Hall system in presence of tunneling [5] should be especially emphasized. The latters could not be associated with Goldstone mode because the presence of tunneling (nondiagonal term) breaks down the symmetry explicitely and Goldstone mode disappears. Besides that, Goldstone mode kink-solitons do not carry topological charge unlike sine-Gordon kinks. Due to this we expect that they interact elastically with other localizations and define diffusionless transport behaviour in the system like it happens in similar systems with symmetries, particularly, in case of quasi one dimensional antiferromagnets and Fermi-Pasta-Ulam chains of anharmonic oscillators [4].

The phenomelogical model for isolated double layer quantum Hall (pseudo)ferromagnet in the absence of tunneling is effectively described by the following Hamiltonian [3]:

$$\mathcal{H} = \frac{\rho_E}{2}\left(\frac{\partial \mathbf{n}}{\partial x}\right)^2 + \frac{\rho_A - \rho_E}{2}\left(\frac{\partial n_z}{\partial x}\right)^2 + \beta (n_z)^2, \qquad (1)$$

where $\mathbf{n}(x,t)$ is an order parameter unit vector [$n_z(x,t)$ has a meaning of local charge imbalance between the layers]; ρ_A and ρ_E are out of plane and in-plane pseudospin stiffnesses and β gives a hard axis anisotropy.

The time-space behavior of ordering vector could be described in terms of Landau-Lifshitz equation:

$$\frac{\partial \mathbf{n}}{\partial t} = \mathbf{n} \times \mathbf{H}_{eff}, \quad \mathbf{H}_{eff} = 2\left\{\frac{\partial}{\partial x}\left[\frac{\partial \mathcal{H}}{\partial \frac{\partial \mathbf{n}}{\partial x}}\right] - \frac{\partial \mathcal{H}}{\partial \mathbf{n}}\right\} (2)$$

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where H_{eff} is the effective (pseudo)magnetic field. Then from (2) one can write down the motion equations explicitly as

$$\frac{\partial n^+}{\partial t} = 2i\left[2\beta n^z - \rho_A \frac{\partial^2 n^z}{\partial x^2}\right] n^+ + 2i\rho_E n^z \frac{\partial^2 n^+}{\partial x^2}$$
$$\frac{\partial n^z}{\partial t} = i\rho_E \frac{\partial}{\partial x}\left[n^+ \frac{\partial n^-}{\partial x} - n^- \frac{\partial n^+}{\partial x}\right], \quad (3)$$

where the following definition is used $n^{\pm} = n_x \pm i n_y$.

The set of equations (3) permits the uniform solution in the form:

$$n^+ = n_\perp^0 e^{4i\beta n_z^0 t}; \quad n_z = n_z^0; \quad n_\perp^0 = \sqrt{1-(n_z^0)^2}, \quad (4)$$

Preparing initially the sample with a given charge imbalance n_z^0 this quantity will stay unchanged for infinitely long time period as far as the isolated double layer systems are considered and tunneling does not exists.

Let us seek for the solution of (3) as a perturbation of uniform solution (4):

$$n^+ = e^{4i\beta n_z^0 t}(n_\perp^0 + m^+); \qquad n_z = n_z^0 + m_z \quad (5)$$

presenting $\mathbf{m}(x,t)$ in the form of multiple scale expansion according to the general approach [6]:

$$\mathbf{m} = \sum_{\gamma=1}^{\infty} \varepsilon^\gamma \mathbf{m}^{(\gamma)}(\xi,\tau), \quad \xi = \varepsilon(x - vt), \quad \tau = \varepsilon^3 t \quad (6)$$

where ξ and τ are the slow space-time variables, v is a propagation velocity of nonlinear wave and ε is a formal small parameter indicating the smallness or "slowness" of the variable before which it appears. Considering weakly nonlinear limit (perturbative solution) we have a following restriction $(\mathbf{m})^2 \ll 1$.

Building the perturbative solution we substitute Exps. (5) and (6) into the motion equation (3) collecting the terms of the same order over ε. Using the general approach [6] we finally come to the Korteweg - deVries equation obtaining thus one soliton solution in the leading approximation:

$$n^+ = n_\perp^0 - i\varphi; \qquad n_z = n_z^0 + \sqrt{\frac{\rho_E}{2\beta}} \cdot \frac{\partial \varphi}{\partial x} \quad (7)$$

where φ is a real function:

$$\varphi = A \tanh\left[\frac{x - vt}{\Lambda}\right], \quad (8)$$

where $A \ll 1$ is an amplitude of weakly nonlinear wave; Λ is a soliton width

$$\Lambda = \sqrt{\frac{2\rho_E}{\beta}\frac{(n_\perp^0)^2\rho_A + (n_z^0)^2\rho_E}{n_z^0 \rho_E A}}; \quad (9)$$

and v is a propagation velocity of the kink-soliton coinciding with the velocity of linear Goldstone mode $v = n_\perp^0\sqrt{8\beta\rho_E}$ (see e.g. Ref. [3]).

As it is seen from Eqs. (7) and (8) this localized object has a kink like form in pseudospin xy plane while it is ordinary moving soliton considering the z component of pseudospin. From the same expressions we also see that in fully balanced case $n_z^0 \to 0$ the kink-soliton solution disappears - its localization width Λ and excitation amplitude go to infinity and zero, respectively. Other marginal case is strongly unbalanced situation $n_z^0 \to 1$, i.e. almost all charge is distributed in one of the layers. In this case $v \sim n_\perp^0 \to 0$ and the used type of multiple scale analysis is not applicable and separate consideration is needed which has been done in Ref. [7].

Thus we can conclude that for balanced situation and nonzero distance between the layers Goldstone mode induces the acoustic branch of linear excitations. While in unbalanced state $n_z^0 \neq 0$ only excitations in the system are Goldstone mode kink-solitons.

Considered kink-solitons are moving charged localizations and therefore could be easily detected experimentally. Indeed, any perturbation of unbalanced double layer system (which could be caused by temporary application of local electric field perpendicular to the layers plane) will induce inhomogeneous electric current through each layer.

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References

[1] I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, K. W. West, Phys. Rev. Lett., **87**, 036803 (2001).

[2] H. A. Fertig, Phys. Rev. B, **40**, 1087, (1989).

[3] K. Yang, et. al., Phys. Rev. B, **54**, 11644, (1996); L. Balents, L. Radzihovski, Phys. Rev. Lett., **86**, 1825, (2001);

[4] L.D. Faddeev, L.A. Takhtajan, Phys. Lett., **85A**, 375, (1981); R. Khomeriki, Phys. Rev. E, **65**, 026605, (2002).

[5] J. Kyriakidis, at. al., Phys. Rev. Lett., **83**, 1411, (1999).

[6] T. Taniuti, Progr. Theor. Phys. (Suppl.), **55**, 1, (1974); N. Giorgadze, R. Khomeriki, Physica B, **252**, 274, (1998).

[7] R. Khomeriki, M. Abolfath, K. Mullen, Phys. Rev. B, **65**, 121310(R), (2002).