Effects of disorder on conductance through small interacting systems

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Abstract

We study the effects of disorders on the transport through small interacting systems based on a two-dimensional Hubbard cluster of finite size connected to two noninteracting leads. This system can be regarded as a model for the superlattice of quantum dots or atomic network of the nanometer size. We calculate the conductance at $T = 0$ using the order U^2 self-energy in an electron-hole symmetric case. The results show that the conductance is sensitive to the randomness when the resonance states are situated near the Fermi energy.

Key words: quantum transport; mesoscopic system; electron correlation; disorder; Hubbard model; two dimension

Quantum transport through small interacting systems, such as quantum dots and wires, has been a subject of current interest. For these systems theoretical approaches, which are able to treat correctly the interaction and interference effects, are necessary for systematic investigations. In this report, using the perturbation method described in [1], we study effects of disorders on the conductance of the interacting systems.

We consider a system consisting of three regions; a finite interacting region at the center (C), and two noninteracting reservoirs on the left (L) and right (R). The total Hamiltonian is, $\mathcal{H}_{\rm tot} = \mathcal{H}^0 + \mathcal{H}_{\rm C}^{\rm int}$, with $\mathcal{H}^0 = \mathcal{H}_{\rm L} + \mathcal{H}_{\rm R} + \mathcal{H}_{\rm C}^0 + \mathcal{H}_{\rm mix}$. Here $\mathcal{H}_{\rm L}$ ($\mathcal{H}_{\rm R}$) is a Hamiltonian for the left (right) lead. The central region is described by a Hubbard model on a square lattice of $N \times M$ ($= N_{\rm C}$) sites: $\mathcal{H}_{\rm C}^0 = -\sum_{jj'\in {\rm C}} t_{jj'}^{\rm C} c_{j\sigma}^\dagger c_{j'\sigma}$, and $\mathcal{H}_{\rm C}^{\rm int} = U \sum_{j=1}^{N_{\rm C}} [\, n_{j\uparrow}\, n_{j\downarrow} - (n_{j\uparrow} + n_{j\downarrow})/2 \,]$ in the standard notation. In this report we consider the electron-hole symmetric case, and examine effects of the off-diagonal disorder described the nearest-neighbor transfer $t_{jj'}^{\rm C}$. The central region and two leads are connected via M channels described by $\mathcal{H}_{\rm mix}$ [1]. We assume that the two coupling are equal and described by a parameter $\Gamma = \pi\rho\, v^2$, where v is the mixing matrix element and ρ is the density of states of the isolated leads. The system may be regarded as a model for the superlattice of quantum dots [2].

The zero-temperature conductance of this system is determined by the value of the single-particle Green's function at the Fermi energy $\omega = 0$ [1]. The Dyson equation is written in a $N_{\rm C} \times N_{\rm C}$ matrix form

$$\left\{\widehat{\boldsymbol{\mathcal{G}}}(z)\right\}^{-1} = \left\{\widehat{\boldsymbol{\mathcal{G}}}^{\,0}(z)\right\}^{-1} - \widehat{\boldsymbol{\Sigma}}(z), \tag{1}$$

$$\left\{\widehat{\boldsymbol{\mathcal{G}}}^{\,0}(z)\right\}^{-1} = z\,\widehat{\mathbf{1}} - \widehat{\boldsymbol{\mathcal{H}}}_{\rm C}^{\,0} - \widehat{\boldsymbol{\mathcal{V}}}_{\rm mix}(z)\,. \tag{2}$$

Here $\widehat{\boldsymbol{\mathcal{G}}}^{\,0} = \{G_{jj'}^0\}$ with $jj' \in {\rm C}$ is the unperturbed Green's function corresponding to $\mathcal{H}^0$, and $\widehat{\boldsymbol{\Sigma}} = \{\Sigma_{jj'}\}$ is the self-energy due to the interaction $\mathcal{H}_{\rm C}^{\rm int}$. We assume the hard-wall boundary condition for $\widehat{\boldsymbol{\mathcal{H}}}_{\rm C}^{\,0} = \{t_{jj'}^{\rm C}\}$ along the direction perpendicular to the current. The size along this direction is M. The mixing self-energy $\widehat{\boldsymbol{\mathcal{V}}}_{\rm mix}$ is non-zero only for the two subspaces corresponding to the interfaces, for each of which the partitioned matrix is given by $-i\Gamma\mathbf{1}$ for the retarded function $z = \omega + i0^+$ with $\mathbf{1}$ being the unit matrix of size M. We calculate the value of $\widehat{\boldsymbol{\mathcal{G}}}$ at $\omega = 0$ using eq. (1) with the order U^2 self-energy $\widehat{\boldsymbol{\Sigma}}(0)$ as in [1]. In the present study, we take the nearest-neighbor transfer to be random variables between $0.9 < t_{jj'}^{\rm C}/t < 1.1$, and take the strength of the mixing to be $\Gamma/t = 0.75$, where t is the transfer for the regular cluster.

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We show in the following the results obtained for the system of the size $M = 4$ and $N = 4$. In Fig. 1 the conductance for 29 different samples of random configurations (dashed lines) and that of the regular cluster without the disorder (solid line) are plotted vs U. The conductance for each of these samples decreases with increasing U. Due to the randomness, the value of the conductance fluctuates around that of the regular cluster. To see these features of the results from a different viewpoint, we evaluate the eigenvalues of an effective Hamiltonian $\widehat{\boldsymbol{\mathcal{H}}}_{\rm C}^{\rm eff} \equiv \widehat{\boldsymbol{\mathcal{H}}}_{\rm C}^{0} + \widehat{\boldsymbol{\Sigma}}(0)$. The eigenvalues can be related to the peak position of the resonance states [1], and among $N_{\rm C}$ eigenstates those near the Fermi energy $\omega = 0$ contribute to the transport. In Fig. 2 the eigenstates near $\omega = 0$ are compared to those for the regular cluster without disorder (dashed lines), where the sample #1 (#2) is a typical example, the conductance of which is larger (smaller) than that of the regular cluster. Note that the eigenvalues are symmetric with respect to $\omega = 0$ due to the electron-hole symmetry, and thus the four eigenvalues shown in each figure are classified into two pairs. In both of the samples the pair situated closer to the Fermi energy keep staying near $\omega = 0$ for $U/(2\pi t) \lesssim 2.0$, while the other pair leave away from the Fermi energy as U increases. Therefore, for the results of the conductance shown in Fig. 1 the contribute of the pair at the near side is about $2\,(2e^2/h)$ and almost independent of the value of the onsite repulsion for small $U/(2\pi t)$. Thus, the other pair at the far side mainly determine the U dependence of the conductance of the cluster examined here.

This feature can be understood from the electronic structure of the regular cluster for $U = 0$, where the system has a well-defined subband structure. In this limit the pair at the far side correspond to the resonant states in the lowest and highest subbands, which are situated near the edge of the subbands. Since in each of these two subbands the Fermi energy is also close to the band edge, the position of the resonance states relative to the Fermi energy becomes sensitive to the disorder and interaction. The contributions of the conducting subbands at the marginal positions play an important role for the fluctuation of the current through the systems with a small number of conducting channels. Details of the formulation, numerical results and discussions will be presented elsewhere.

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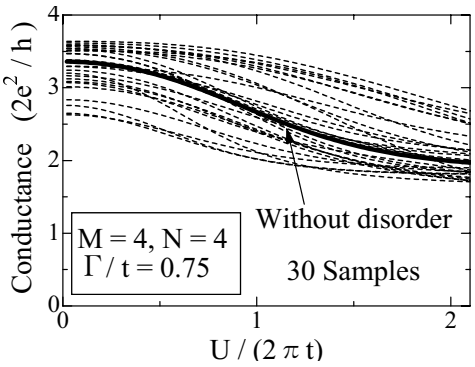

Fig. 1. Conductance of a Hubbard cluster of 4×4 at half-filling with the off-diagonal randomness $0.9 < t'/t < 1.1$. Disordered samples (29 dashed lines), and regular cluster (solid line).

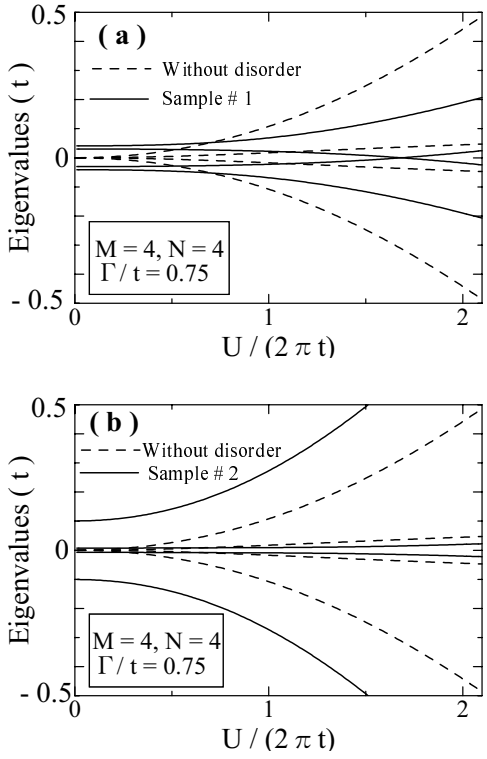

Fig. 2. Eigenvalues of the effective Hamiltonian $\widehat{\boldsymbol{\mathcal{H}}}_{\rm C}^{\rm eff}$ near the Fermi energy $\omega = 0$, which are related to the resonance states contributing to the current.

References

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