

# The peak in the nonlinear ac resistivity of granular superconductors

Mai Suan Li<sup>a</sup>, Hoang Zung<sup>b,1</sup>, D. Domínguez<sup>c</sup>

<sup>a</sup>*Institute of Physics, Polish Academy of Sciences, Aleja Lotników 32/46, 02-668 Warsaw, Poland*

<sup>b</sup>*Vietnam National University, 227 Nguyen Van Cu, Ho Chi Minh City*

<sup>c</sup>*Centro Atómico Bariloche, 8400 San Carlos de Bariloche, Rio Negro, Argentina*

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## Abstract

We model *s*-wave and *d*-wave disordered granular superconductors with a three-dimensional random network of Josephson junctions with finite self-inductance. The nonlinear ac resistivity  $\rho_2$  was calculated numerically. We find a peak in  $\rho_2$  as a function of temperature, in good agreement with recent experiments. The value of  $\rho_2$  at the peak temperature  $T_p$  depends on the current amplitude  $I_0$  as a power law,  $\rho_2(T_p) \sim I_0^{-\alpha}$ . We find that  $\alpha$  depends on the self-inductance and current regimes. In the weak current regime is  $\alpha = 0.5 \pm 0.1$  and independent of the self-inductance for both of *s*- and *d*-wave materials. In the strong current regime,  $\alpha$  depends on the screening, with  $\alpha \approx 1$  for some interval of inductance in agreement with measurements in *d*-wave high  $T_c$  ceramic superconductors.

*Key words:* granular superconductors; pi junctions; *d*-wave superconductivity; Josephson networks

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Recently, Yamao *et al.*[1] have measured the ac linear resistivity  $\rho_0$  and the nonlinear resistivity  $\rho_2$  of ceramic superconductor  $\text{YBa}_2\text{Cu}_4\text{O}_8$ .  $\rho_2$  is defined as the third coefficient of the expansion of the voltage  $V(t)$  in terms of the external current  $I_{ext}$  as  $V = \rho_0 I_{ext} + \rho_2 I_{ext}^3 + \dots$ . When the sample is driven by an ac current  $I_{ext}(t) = I_0 \sin(\omega t)$ , one can obtain  $\rho_2$  from

$$\rho_2 = -\frac{4V'_{3\omega}}{I_0^3}, \quad V'_{3\omega} = \frac{1}{2\pi} \int_{-\pi}^{\pi} V(t) \sin(3\omega t) d(\omega t). \quad (1)$$

Yamao *et al.* have found that  $\rho_2$  has a maximum value at a temperature  $T_p$  near the intergrain ordering temperature of their sample. They observed that  $\rho_2$  depends with  $I_0$  as  $\rho_2(T_p) \sim I_0^{-\alpha}$ , with  $\alpha \approx 1.1$ .

It is now believed that the gap of high- $T_c$  superconductors has *d*-wave symmetry. This makes possible to have weak links with negative Josephson coupling between the superconducting grains in high- $T_c$  ceramics, which are called  $\pi$ -junctions [2]. Therefore, they can be modeled with a network of Josephson junctions with random couplings, given by the hamiltonian [3–5]

<sup>1</sup> E-mail:dung@hcmuns.edu.vn

$$H = - \sum_{\langle ij \rangle} J_{ij} \cos(\theta_i - \theta_j - A_{ij}) + \frac{1}{2L} \sum_p \Phi_p^2. \quad (2)$$

Here  $\theta_i$  is the superconducting phase of the grain at the  $i$ -th site of a cubic lattice,  $J_{ij}$  is the Josephson coupling between grains, and  $L$  is the self-inductance of a loop (mutual inductances are neglected). The first sum is taken over all nearest-neighbor pairs and the second sum is taken over all elementary plaquettes on the lattice. The total magnetic flux threading through the  $p$ -th plaquette is  $\Phi_p = \frac{\phi_0}{2\pi} \sum_{\langle ij \rangle} A_{ij}$  with  $A_{ij} = \frac{2\pi}{\phi_0} \int_i^j \mathbf{A}(\mathbf{r}) d\mathbf{r}$ . We model the *d*-wave superconducting case by taking  $J_{ij}$  as a random variable equal to  $J$  or  $-J$  with equal probability (representing 0 and  $\pi$  junctions respectively), and also the *s*-wave superconducting case by taking  $J_{ij} > 0$  and uniformly distributed in  $[0, 2J]$ . The effect of screening currents is characterized by the dimensionless inductance  $\tilde{L} = (2\pi/\Phi_0)^2 L J$ . The *d*-wave model has been able to reproduce the paramagnetic Meissner effect [3] observed experimentally in ceramic high- $T_c$  cermamics [6]. Kawamura [4,5] proposed that there is a chiral glass phase, which has been seen experimentally in the nonlinear ac magnetic susceptibility [7] and in the aging phenomenon [8].

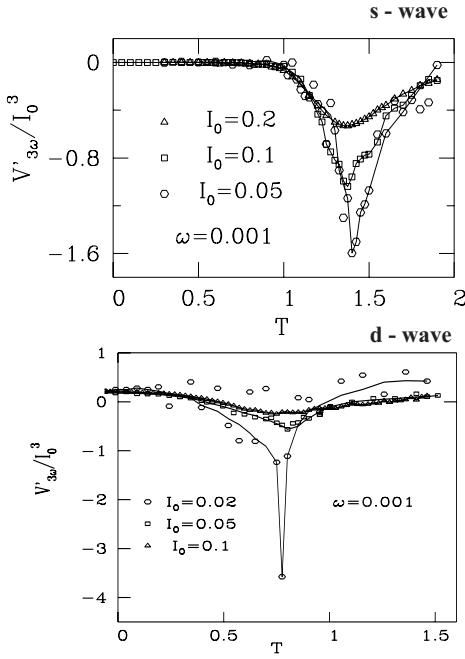


Fig. 1. Temperature dependence of the nonlinear resistivity  $\rho_2 \propto V'_{3\omega}/I_0^3$  for the *s*-wave (upper panel) and the *d*-wave case (lower panel) for  $\tilde{L} = 1$ ,  $\omega = 0.001$  and  $8 \times 8 \times 8$  samples.

To calculate transport properties we use the resistively shunted junction model in which the dissipative ohmic current due to an intergrain resistance  $R$  and the temperature dependent Langevin noise current are added to the Josephson current [3]. This leads to a set of dynamical equations for  $\theta_i$  and  $A_{ij}$  [3], which are solved numerically for a given temperature  $T$  and driving current  $I_{ext} = I_0(\sin \omega t)$  [9,10]. The temperature dependence of the *nonlinear* resistivity  $\rho_2$  for different values of  $I_0$  is shown in Fig.1 for the *s*-wave system (upper panel) and for the *d*-wave system (lower panel) for  $\tilde{L} = 1$  and  $\omega = 0.001$ . We find that in both cases  $\rho_2(T)$  has a peak at a temperature  $T_p$  (different for each case). In the *s*-wave case,  $T_p$  coincides with the metal–superconductor transition at which the linear resistivity vanishes [10], while for the *d*-wave case  $T_p$  coincides with the temperature for the onset of the paramagnetic Meissner effect [9]. The maximum value of  $\rho_2$  at  $T_p$  tends to diverge when  $I_0$  decreases. We have studied [9,10] the dependence of  $\rho_2(T_p)$  with  $I_0$  for the two cases and for different values of  $\tilde{L}$ . We have clearly distinguished two different regimes of power law behavior for small and large currents [10]. In the weak current regime ( $I_0 \leq 0.1$ ) we can fit a dependence  $\rho_2(T_p) \sim I_0^{-\alpha}$ . We obtain  $\alpha = 0.5 \pm 0.1$  independently of the value of  $\tilde{L}$  and both for the *s*-wave and for the *d*-wave cases. In the strong current regime (SCR) we can fit a different exponent  $\alpha$ . However, the value of  $\alpha$  in this regime is different in the *s*-wave and *d*-wave case and

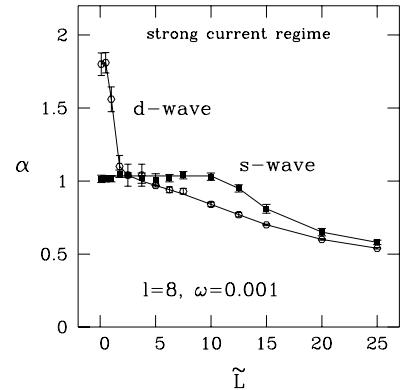


Fig. 2. Dependence of the power-law exponent  $\alpha$  with self-inductance  $\tilde{L}$  obtained in the strong current regime for  $\omega = 0.001$  and  $8 \times 8 \times 8$  samples.

depends strongly on  $\tilde{L}$ , as it is shown in Fig. 2.

In conclusion, we have calculated the non-linear ac resistivity exponent  $\alpha$  for *s* and *d*-wave granular superconductors, obtaining two distinct current regimes. For weak currents  $\alpha$  is independent of the screening strength and of types of pairing symmetry, while in the opposite case this exponent depends on  $\tilde{L}$ . Since real current is  $I = \frac{2eJ}{h}I_0$ , and  $J \sim 10^2$  K, then for  $I_0 \sim 0.1$  we have  $I \sim 10^{-2}$  mA. The experiments of [1] used a current  $I \sim 10$  mA. This suggests that they were performed in the SCR. A typical value of inductance for ceramics is  $\tilde{L}$  are bigger than 3 [11]. As seen from Fig. 2, the value of  $\alpha$  in the SCR for  $1 < \tilde{L} < 5$  agrees very well with the experimental value.

## References

- [1] T. Yamao *et al.*, J. Phys. Soc. Jpn **68** (1999) 871.
- [2] M. Sigrist, T. M. Rice, J. Phys. Soc. Jpn. **61** (1992) 4283.
- [3] D. Domínguez *et al.*, Phys. Rev. Lett. **72** (1994) 2773.
- [4] H. Kawamura, J. Phys. Soc. Jpn **64** (1995) 711 .
- [5] H. Kawamura, M.S. Li, Phys. Rev. B **54** (1996) 619; Phys. Rev. Lett. **78** (1997) 1556 ; J. Phys. Soc. Jpn (1997) **66**, 2110 .
- [6] P. Svelindh *et. al.*, Physica C **162-164** (1989) 1365 ; W. Braunisch *et. al.*, Phys. Rev. Lett. **68** (1992) 1908.
- [7] M. Matsuura *et al.*, J. Phys. Soc. Jpn **64** (1995) 4540 .
- [8] E. L. Papadopoulou *et al.*, Phys. Rev. Lett. **82** (1999) 173; M. S. Li *et al.*, Phys. Rev. Lett. **86** (2001) 1339.
- [9] M. S. Li and D. Domínguez, Phys. Rev. B **62**, 14554 (2000).
- [10] M. S. Li, H. Zung and D. Domínguez, Phys. Rev. Lett. (2002).
- [11] R. Marcon *et al.*, Phys. Rev. **39** (1989) 2796 .