Frequency noise in hysteretic resistive-SQUIDs

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Abstract

We present measurements of hysteretic resistive SQUIDs (R-SQUIDs) as a function of the hysteresis parameter $\beta_L = 2\pi L I_c/\Phi_0$. Varying β_L *in situ* from 15 to 45, we observe quasiperiodic peaks in frequency noise which *decrease* toward the thermal limit with *increasing* temperature. We have developed a model illustrating the dynamics of the two characteristic time scales: Josephson phase oscillations and catastrophic transitions in phase. From this model, an intuitive energy picture emerges wherein low thermal energy is shown to promote trapping in false minima leading to excess frequency noise. The energy model further illustrates that confusion between degenerate branch solutions results in quasiperiodic noise peaks.

Key words: SQUID; R-SQUID; resistive-SQUID; hysteresis; frequency noise

1. Introduction

The core element of many practical superconducting devices is a Josephson junction. While the junction behavior in isolation is quite simple, though nonlinear, the behavior of any superconducting circuit is critically dependent upon the elements to which the junction is coupled. One of the simplest circuits is a junction shunted by a resistor, forming a loop, which is then powered by an external dc source. This circuit is known as a resistive-SQUID, or R-SQUID. Its behavior is smooth and analytically soluble as long as the junction's critical current is sufficiently small that the inductance of the loop does not cause the hysteresis parameter $\beta_L = 2\pi L I_c/\Phi_0$ to exceed unity. When driven by a fixed current, this circuit can be used as a clean (though nonsinusoidal) frequency source.

In contrast, we have measured the frequency noise of this simple circuit in the case where β_L is significantly greater than unity. We find that this noise is considerably larger than the thermal limit, *decreases* with increasing temperature, and varies significantly with, and quasiperiodically in, β_L.

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2. Experimental Observations

Our R-SQUIDs typically have zero temperature critical currents of $300\mu A$, shunting resistance of $10\mu\Omega$, and inductance of 40 pH. The junction's critical current was also adjustable *in situ* by a control line running directly under it. All of the superconducting circuits required for our measurements (R-SQUID, cou-

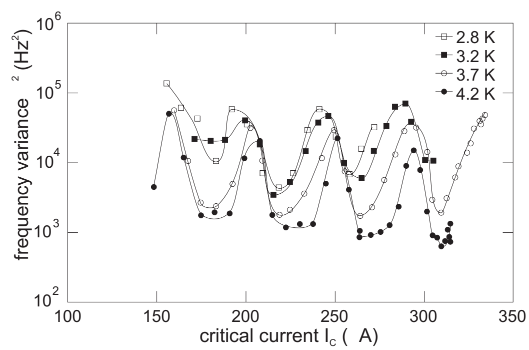

Fig. 1. Frequency variance of an R-SQUID oscillator as a function of I_C (proportional to β_L) at several different temperatures. The two features to note are the quasiperiodic oscillation with β_L and the increasing frequency noise with decreasing temperature.

pling network, preamp, and output amplifier) are integrated together on a single chip. Details of their design and construction are deferred to Ref. [1] except to note that they were fabricated by TRW's LTS Foundry using their JJ-110D niobium-based process [2].

We measured the frequency of the R-SQUID at a fixed driving current ($I > I_C$) by counting cycles in 0.1 sec intervals. This frequency was qualitatively consistent with that predicted by a simple model[3], but could not be reconciled with it quantitatively over a range of driving currents or β_L.

The variance in repeated measurements of this frequency is plotted in Fig. 1. Contrary to expectations, the frequency noise was strongly dependent upon β_L and *decreased* with increasing temperature, approaching the thermal limit only in certain intervals of β_L.

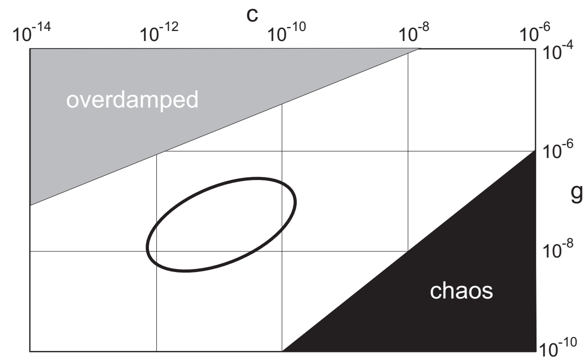

Fig. 2. Phase diagram of complete R-SQUID model showing the different characters of behavior in the low-inertia (high damping) limit and in the underdamped (chaotic) limits. The devices we have studied experimentally are in the region indicated by the ellipse.

3. Theoretical Model

Although it is not obvious *a priori*, many of the features of our results can be understood in the simple model which considers the junction to be defined completely by the Josephson relation, $I = I_c sin\phi$. Details of the junction such as its capacitance and sub-gap conductance turn out to be unimportant during almost the entire evolution of the circuit's behavior. This model results in a first-order differential equation which can be integrated analytically to give the junction current as a function of time.[3]

This result is correct for a non-hysteretic ($\beta_L < 1$) R-SQUID, but when β_L exceeds unity, smooth evolution of the junction's phase requires a loop backwards in time. For large enough β_L several different loops will overlap. In this simple model, at a critical junction phase, ϕ_c, the junction suffers a catastrophic transition to a new phase. Plots of the solutions to the first-order model[3] might suggest the final phase after the transition, but such suggestions are incorrect.

In the limit that the transition time is short compared to the inverse Josephson frequency (a circumstance well obeyed by our circuit parameters) the final phase is constrained by the observation that the voltage across the R-SQUID's resistor is bounded, and hence in the limit its integral through the transition is zero. This integral is proportional to the function $f(\phi)$ first introduced by McCumber[4] as $f(\phi) = \phi + \beta_L \sin\phi$. The final phase after a catastrophic transition, ϕ_f, is then constrained by the condition $f(\phi_f) = f(\phi_c)$.

The first-order model, however, cannot distinguish between several possible branches, because on each one $f(\phi)$ can be conserved. For this, we need a more complete model incorporating both the junction's sub-gap conductance, G_J, and intrinsic capacitance, C. For our devices, these quantities were originally neglected because $g \equiv G_J R \approx 10^{-8}$ and $\beta_C \equiv 2\pi I_C R^2/\Phi_0 \approx 10^{-12}$. These quantities *can* be neglected for all times *except* in the immediate vicinity of the catastrophe, in which case the full model needs to be solved.

Over a range of the parameters G_J and C we observe a variety of behaviors as indicated in Fig. 2, including chaotic time evolution as studied by several other groups.[5] An intuitive picture predicting the level of frequency noise observed has emerged from this work. In this picture, quasi-periodic frequency noise arises from "confusion," the difficulty near certain values of β_L in determining the optimum branch solution following a catastrophe. Thermal noise, paradoxically, aids in this determination, reducing the level of frequency noise as temperature increases.

Acknowledgements

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