Quantum phase transition of striped domain walls

Tsutomu Momoi [a],[1]

[a] *Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan*

Abstract

Quantum melting of 2D striped domain wall arrays are investigated at zero temperature. It is revealed that the stripes with short-range interactions melt and become a stripe liquid in low density due to dislocations created by quantum fluctuation. This quantum melting transition is of second order and in the 3D XY universality class. Quantum melting in the presence of long-range interactions that fall off as power laws is also studied. This result is applied to incommensurate domain walls in 2D adsorbed atoms on substrate and doped antiferromagnets.

Key words: Stripes; incommensurate phase; stripe liquid; quantum melting

1. Introduction and Summary

Recently, striped domain walls with strong quantum fluctuations were attracting extensive interests, because incommensurate stripe structures, whose order is short ranged, were observed in doped nickel oxides and cupper oxides[1]. Striped domain walls were also observed in two-dimensional (2D) solids of adsorbed atoms on graphite[2], where quantum fluctuations are large for light atoms, e.g. ^{3}He and H_2. Striped domain walls were also found in a 2D quantum spin system.[3] However, effects of quantum fluctuation on the 2D domain-wall structures are hardly understood and, in theoretical understanding, little is known about quantum disordered stripe states.

In this paper, we study quantum fluctuations in stripes using the elastic theory and taking account of dislocations at zero temperature. It is shown that in low density the striped structure with short-range wall-wall interactions is unstable against quantum fluctuations and becomes a stripe liquid even at zero temperature.[4] This melting transition is in the 3D XY universality class. On the other hand, if stripes have long-range wall-wall interactions that fall off as power laws r^{-q+1} with $2 < q < 7$, the stripe structure is stable for low density and unstable for high density. This tendency to ordering is opposite to the case of short-range interaction systems.

We note that the striped ordered (solid) structure can be observed as sharp incommensurate Bragg peaks. On the other hand, in the stripe disordered (liquid) state, incommensurate scattering peaks have finite width, which means finite correlation length. Comparing with experiments done for adsorbates and cuprates, we point out that the quantum disordered stripes were already observed in scattering experiments of adsorbed H_2/Gr and D_2/Gr,[5] and doped cuprates[6], e.g. $La_{2-x}Sr_xCuO_4$, in which the correlation length is much smaller than the system size.

2. Mechanism of Quantum Melting Transition

It has been argued for doped antiferromagnets[7] that a single domain wall behaves like a quantum elastic string with wall mass m per unit length and interfacial stiffness γ. In adsorbed atom systems, domain walls are not pinned by substrate periodic potential, but floating, and hence we also adopt this string model. Let us consider striped domain walls with spacing l aligned along the y-axis. Parallel domain walls interact with each other by both exponential repulsion and

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[1] E-mail:momoi@cm.ph.tsukuba.ac.jp

hard-core potential, and hence stiffness (K_x) appears between walls in x-direction. In the continuum limit, the effective action for stripes without any dislocation is[7]

$$S_0[u] = \frac{K}{2\hbar} \int dT d^2 R \{ (\partial_T u)^2 + (\partial_X u)^2 + (\partial_Y u)^2 \} \quad (1)$$

with $K = (mK_x/l)^{1/2}$, where we have rescaled as $T = \sqrt{\gamma/m}\tau$, $X = (\gamma/lK_x)^{1/2}x$, and $Y = y$. Here u denotes displacement of the wall.

We next consider the singular part in u that comes from dislocations. In the 2+1D space, the dislocations become lines (or loops), which can be observed[8] by a loop integral on a closed loop Γ

$$\oint_\Gamma du = Qls, \quad (2)$$

where s denotes the number of dislocation lines enclosed by the loop Γ and Q degeneracy of domain walls. Q domain walls merge into a dislocation so that each dislocation is consistent with the domain structure. The field u can be divided into the smooth part u_{smooth} and the singular part. We thus obtain the total effective action

$$S = S_0[u_{\text{smooth}}] + S_D[\mathbf{n}] \quad (3)$$

with

$$S_D[\mathbf{n}] = \frac{1}{2\hbar} \int \frac{d^3 q}{(2\pi)^3} \frac{(Ql)^2 K}{q^2} |\mathbf{n}(\mathbf{q})|^2, \quad (4)$$

where $\mathbf{q} = (q_X, q_Y, q_T)$ and $\mathbf{n}(\mathbf{q})$ denotes Fourier transform of dislocation density vectors.

After transforming as $\tilde{u} = 2\pi u/Ql$, the system has periodicity with 2π and the effective action is equivalent to a spatially anisotropic 3D XY model in the vortex loop representation.[8] We thus mapped the 2D quantum stripe system with dislocations onto the anisotropic 3D XY model with couplings $J_\tau = J_y = Q(Ql)^2 \sqrt{m\gamma}/(2\pi)^2$ and $J_x = (Ql)\sqrt{m/\gamma}K_x/(2\pi)^2$.

For the mapped 3D XY model, it is natural to expect that the critical point is given by the relation $(J_x J_y J_\tau)^{1/3}\hbar^{-1} = C$ with a constant C of order unity and the system becomes disordered in $(J_x J_y J_\tau)^{1/3}\hbar^{-1} < C$. Note that the dislocation loops reduce the stiffness to zero in the disordered phase.[8,9] This melting is a continuous transition in the same universality class as the 3D XY model. In the stripe liquid phase, the correlation length behaves as $\xi \sim (n_c - n)^{-\nu}$ with $\nu \approx 0.67$ near the critical density n_c.
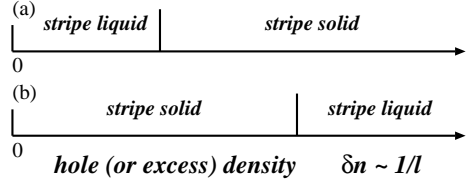

Fig. 1. Phase diagrams of 2D stripes with (a) exponential interaction and (b) power-decay one, u^{-q+1}, with $2 < q < 7$.

3. Phase Diagrams

Using the above melting criterion, we obtain phase diagrams of stripe liquid and ordered phases.

In the case of short-range interactions that fall off in an exponential form, wall fluctuations slightly renormalize the interaction, but do not change the exponential form for all parameter region[10]. Thus the stiffness K_x also decays exponentially. Inserting this into the criterion, we conclude that the stripes melt for large l (in low density) and become a disordered state, i.e., a stripe liquid. The phase diagram of the stripe phase is shown in Fig. 1 for fixed $m < \infty$.

Next consider stripes repelling with each other by power decay wall-wall interaction, u^{-q+1}. (Coulomb repulsion between lines, $-\ln u$, corresponds to $q = 1$.) However, in the case of $q \leq 2$, the plasma modes appear and low-lying excitations in k_x-direction do not have k-linear spectrum. Hence stiffness K_x diverges and the elastic theory cannot be applied to $q \leq 2$ case. Hereafter we consider only the case $q > 2$. In this case, no renormalization appears in the interactions[10]. From the melting criterion, we find that for $2 < q < 7$, stripes stabilize in low density and melt in high density. (See Fig. 1.) This behavior is similar to that of charged point objects, e.g. Wigner crystals. On the other hand, for $q > 7$ stripes become a liquid for low density the same as the short-range interaction systems.

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