Carrier drift velocity in semiconducting strings

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Abstract

The carrier drift velocity in a 1D semiconducting sample attached to electrodes is derived from the Luttinger model, where the electric field generated by image charges within the electrodes is explicitly taken into account. If the length of the electrode is small enough compared to the sample length, the drift velocity v_d can be estimated as $v_d \simeq (4e^2/h)(1/\varepsilon_s)$, where e, h, and ε_s are the elementary charge, the Planck constant, and the dielectric constant of the semiconductor, respectively. Compared to the experimental values of v_d, the obtained relationship for v_d is found to be still valid for a 3D sample in the large limit of the external field.

Key words: Luttinger liquid; Landauer model; metal contact; image charge; large-field limit

It is known that the carrier drift velocity in most (1D) semiconductors tends to be saturated with increasing applied electric field. Up to now, the evaluation of the drift velocity has been mostly based on the (Boltzmann) transport equation. However, under a high field where the drift velocity tends to be saturated, the validity of the (semiclassical) Boltzmann equation has turned out to be violated due to some quantum effects [1]. The aim of this article is, based on the Luttinger model, to estimate the drift velocity of a semiconductor with metal contacts.

We begin with a spinless (bosonized) Luttinger liquid attached to electrodes (at $x = 0, L$, with their length ΔL). Under the electromagnetic field A_ν ($\nu = 0, 1$), the model Lagrangian is given by [2]

$$\mathcal{L} = \frac{h}{4}\frac{v(x)}{K(x)}\left[(\partial_{vt}\phi)^2 - (\partial_x\phi)^2\right] - e\textstyle\sum_{\mu,\nu}\epsilon^{\nu\mu}A_\nu\partial_\mu\phi, \quad (1)$$

where h, $K(x)$, $v(x)$, and e represent the Planck constant, the interaction-dependent parameter characterizing the Luttinger model, the velocity, and the elementary charge, respectively. Here $\epsilon^{\nu\mu}$, which is antisymmetric, is chosen as $\epsilon^{10} = 1$ [with the metric $(+,-)$], and the scalar field ϕ is normalized such that the density ρ and current j are related to $\rho = -\partial_x\phi, j = \partial_t\phi$. Then in a stationary current regime ($\partial_t j = 0$), the field equation for ϕ indicates that

$$\partial_x(v\rho/K) = 2eE/h, \quad (2)$$

where E ($= -\partial_x A_0 + \partial_t A_1$) is the electric field.

Given the metal contacts, E can be composed of the external field $E^{(\mathrm{ex})}$ and the Coulomb field generated by the image charge, $E^{(\mathrm{img})}$, namely

$$E = E^{(\mathrm{ex})} + E^{(\mathrm{img})}. \quad (3)$$

Notice that the direct Coulomb field due to the electron-phonon and electron-electron interactions is taken into account through the parameter $K(x)$. If the image charges are realized only within the electrodes, $E^{(\mathrm{img})}$ can be written using ρ as

$$E^{(\mathrm{img})}(x) = \frac{-e}{\varepsilon_s}\sum_{i=0}^{1}\int_{\Omega_i}\mathrm{sgn}\,(x - x')\frac{\rho(2iL - x')}{(x - x')^2}\,\mathrm{d}x', \quad (4)$$

where ε_s is the dielectric constant of the sample semiconductor, $\Omega_0 = [-\Delta L, 0]\cap[-L, 0], \Omega_1 = [L, L+\Delta L]\cap[L, 2L]$, and $\mathrm{sgn}\,(x)$ is the sign function. In a practical case, the condition of $\Delta L \ll L$ is satisfied, so that $E^{(\mathrm{img})}(x)$ can be approximated as

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$$E^{(\mathrm{img})}(x) = \frac{-e}{\varepsilon_s}\Delta L \sum_{i=0}^{1} \mathrm{sgn}\,(x - iL)\rho(iL)(x - iL)^{-2} \times [1 + O(\Delta L/L)]\,. \tag{5}$$

Now we estimate the electric current I $(= ej)$. Under the boundary condition that the field ϕ outside the metal contacts moves out rightward (for $x > L + \Delta L$) or leftward (for $x < -\Delta L$), I can be derived from Eq. (2) as [3]

$$I = (e^2 K_L/h) \int_\Omega E(x)\,\mathrm{d}x, \tag{6}$$

where $K_L = K(x = L + \Delta L) = K(x = -\Delta L)$, with $\Omega = [-\Delta L, L + \Delta L]$. Considering that Eq. (6) corresponds to the Landauer formula, we can interpret K_L as the transmission coefficient $\mathcal{T}$ for the carrier to penetrate from the metal contact to the sample. Once K_L can be regarded as $\mathcal{T}$, the difference of the quasi-Fermi level, which is $\int_\Omega E(x)\,\mathrm{d}x$, is related to the corresponding potential difference with perfectly conducting metal contacts, which amounts to $\int_\Omega E^{(\mathrm{ex})}\,\mathrm{d}x$, through the following relation (at zero temperature) [4]:

$$\int_\Omega E^{(\mathrm{ex})}\,\mathrm{d}x = (1 - \mathcal{T}) \int_\Omega E(x)\,\mathrm{d}x. \tag{7}$$

Substituting Eq. (7) with $\mathcal{T} \to K_L$ into Eq. (6) and using Eqs. (3) and (5), we can rewrite I (where the spin degree of freedom is taken into account) as

$$I = 2 \times (e^2/h)\,\mathrm{p.v.}\int_\Omega E^{(\mathrm{img})}(x)\,\mathrm{d}x = \frac{2e^3}{h\varepsilon_s}[\rho(L) - \rho(0)] \times [1 + O(\Delta L/L)]\,, \tag{8}$$

where p.v. represents the principal value.

On the other hand, I can be expressed as the product of $e\rho(L + \Delta)$ and the drift velocity v_d at $x = L + \Delta$:

$$I = ev_d\rho(L)[1 + O(\Delta L/L)], \tag{9}$$

so that from Eqs (8) and (9), we obtain

$$v_d = \frac{4e^2}{h\varepsilon_s} = \frac{13.93}{\varepsilon_s} \times 10^7\ \mathrm{cm/s} \quad (\text{for } \Delta L \ll L), \tag{10}$$

where use has been made of $\rho(0) = -\rho(L)$ due to the charge conservation.

Before examining the validity of Eq. (10), we show that v_d for $E^{(\mathrm{ex})} \to \infty$ is invariant under the scaling of the cross section S of the 1D sample as $S \to nS$ ($n = 1, 2 \ldots$). Based on the Landauer model, K_L in Eq. (6) is replaced by $\sum_{c_1,c_2=1}^{N_1,N_2} \mathcal{T}_{c_1c_2}$, where $\mathcal{T}_{c_1c_2}$ represents the transmission probability for the channel mode (c_1, c_2) in the y and z directions. If the carriers are well restricted (within the square well potential with infinite height) in the y and z directions, the mode number turns out to be $N_1 = N_2 = n$. For $E^{(\mathrm{ex})} \to \infty$, the dominant energy for the carrier is the kinetic energy in the x direction, so that the mode-exchange scattering may be neglected, and that the remaining nonvanishing scattering can be mode independent, namely, $\mathcal{T}_{c_1c_2} \simeq \delta_{c_1c_2}\mathcal{T}_{11}$. Thus, I scales as $I \to nI$, with the result that v_d ($\propto I/S$) for $E^{(\mathrm{ex})} \to \infty$ is found to be invariant under $S \to nS$.

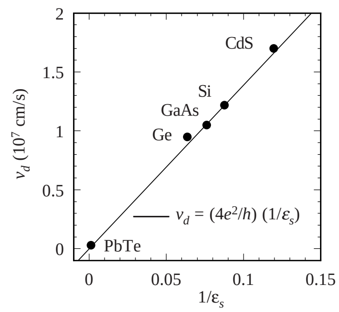

Fig. 1. The (saturation) drift velocity v_d for $E^{(\mathrm{ex})} \to \infty$ vs the dielectric constant ε_s for 3D semiconducting samples at low temperature (4–130K). Notice that for PbTe, the temperature dependence of ε_s [5] and the field dependence of v_d [6] is large enough, compared to those for other samples. The data of ε_s (except PbTe) are taken from Ref. [7], while those of v_d for Si, Ge, CdS, and GaAs are after Refs. [8–11], respectively.

The experimental values of the (saturation) drift velocity v_d for $E^{(\mathrm{ex})} \to \infty$ are shown in Fig. 1. It is found that v_d is well represented by Eq. (10), which indicates the validity of Eq. (10).

In summary, we have estimated, based on the Luttinger model, the drift velocity v_d of a 1D semiconductor whose ends are attached to electrodes. If the image charges are realized only within the electrodes, v_d can be estimated for $\Delta L \ll L$ as in Eq. (10). The obtained relationship for v_d is still valid for a 3D sample in the limit of $E^{(\mathrm{ex})} \to \infty$. The detailed analysis of v_d for $\Delta L \sim L$ is in progress, and will be discussed elsewhere.

Acknowledgements

The authors are indebted to Prof. K. Terada, Dr. M. Terauchi, and T. Okamoto for invaluable comments.

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