Dynamics of vortex lattice formation in a rotating Bose-Einstein condensate

Makoto Tsubota [a,1], Kenichi Kasamatsu [a], Masahito Ueda [b]

[a] *Department of Physics, Osaka City University, Osaka 558-8585, Japan*

[b] *Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

Abstract

We study the dynamics of vortex lattice formation of a rotating trapped Bose-Einstein condensate by numerically solving the two-dimensional Gross-Pitaevskii equation with a dissipative term. The condensate trapped in a quadratic potential forms a triangle lattice of quantized vortices, following the damped elliptic oscillation of the condensate and the excitation of surface waves, which is consistent with the experimental results of an ENS group. A fast rotating condensate confined in a quadratic-plus-quartic potential generates a giant vortex absorbing all phase defects into a single density hole, where a quasi-one-dimensional circular superflow is realized.

Key words: Bose-Einstein condensation; quantized vortex; Gross-Pitaevskii equation

1. Introduction

Recent experimental works have succeeded in observing the vortex lattice formation in the atomic Bose-Einstein condensates (BECs) [1,2]. Motivated by these works, we study the dynamics of a rotating trapped BEC by numerically solving the two-dimensional Gross-Pitaevskii equation that governs the evolution of the order parameter $\psi(\boldsymbol{r}, t)$

$$(\imath - \gamma)\hbar\frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2m} + V + g|\psi|^2 - \mu - \Omega L_z\right]\psi. \quad (1)$$

Here $g = 4\pi\hbar^2 a/m$ is the coupling constant, proportional to the ^{87}Rb scattering length $a \approx$5.77nm. The centrifugal term $-\Omega L_z$ appears in a system rotating about the z axis at a frequency Ω. The dissipation is introduced by the term with γ. The observation of the vortex lattices [1] means the presence of the dissipation, because a vortex lattice corresponds to a local minimum of the total energy in the configuration space [3]. We use $\gamma = 0.03$ in this work. The quadratic trapping potential V makes the condensate develop to the vortex lattice formation consistent with the observations [4]. A fast rotating BEC confined in a quadratic-plus-quartic potential V generate a giant vortex that absorbs all singularities into a single density hole in the center [5].

2. Vortex lattice in a quadratic potential

We first prepare an equilibrium condensate in a stationary potential $V = \frac{1}{2}m\omega_\perp^2(x^2 + y^2)$ with $\omega_\perp = 2\pi \times 219$Hz. The rotation of $\Omega = 0.7\omega_\perp$ starts at $t = 0$ with the anisotropy corresponding to the experiment [1]. Figure 1 shows the typical dynamics of the condensate density $|\psi|^2$ [6]. The condensate is elongated along the x axis because of the small anisotropy of V, and the elliptic cloud oscillates. Then, the boundary surface of the condensate becomes unstable, exciting the surface waves. The ripples on the surface develop into the vortex cores around which superflow circulates. Subject to the dissipative vortex dynamics, some vortices enter the condensate, eventually forming a vortex lattice. As the vortex lattice is being formed, the axial symmetry

[1] E-mail:tsubota@sci.osaka-cu.ac.jp

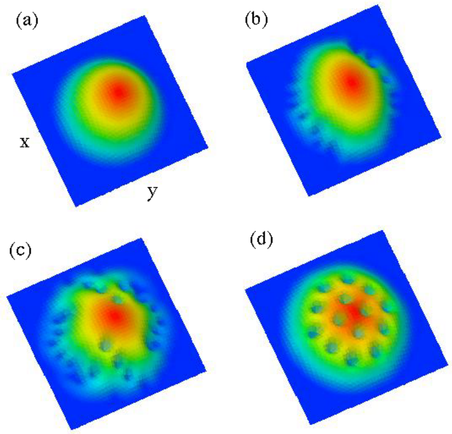

Fig. 1. Time development of the condensate density after rotating the trapping potential.The time is t =0 msec (a), 114 msec (b), 123 msec (c), and 262 msec (d).

of the condensate is recovered by transferring angular momentum into quantized vortices.

This peculiar dynamics is understood better by investigating the phase of ψ. As soon as the rotation starts, the defects of the phase appear on the outskirts of the condensate where the amplitude $|\psi|$ is almost negligible. These defects come into the boundary surface of the condensate within which the density grows up, compete with each other and excite the above surface waves. There the selection of the defects starts, because their further invasion costs the energy and the angular momentum. Some vortices enter the condensate and complete a lattice dependent on Ω.

3. Giant vortex in a combined potential

The argument now extends to a fast rotating regime [7]. Competing with the centrifugal potential in the limit $\Omega \to \omega_\perp$, however, only the quadratic potential can make the research difficult. Thus we use the quadratic-plus-quartic potential $V = (1/2)m\omega_\perp^2 r^2 + (1/4)k(m^2\omega_\perp^3/\hbar)r^4$, studying the dynamics of the condensate after the rotation starts [5].

The effective trapping potential with the centrifugal potential features a 'Mexican hat' structure, when $\Omega > \omega_\perp$. As a result, there appears the central region of low density which absorbs the defects, as shown in Fig. 2 (a) and (b). As Ω is increased further, the hole absorbs all vortices, making a giant superfluid vortex with circular superflow(Fig.2 (c) and (d)). The amplitude of the circular superflow with radius R is given by $v_s = \kappa n/2\pi R$, where n is the number of defects inside the hall and κ the quantized circulation. It should be noted that the phase defects do not overlap completely

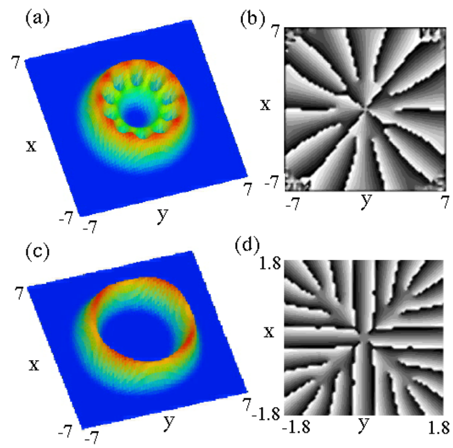

Fig. 2. Density((a) and (c)) and the phase profiles((b) and (d)) under the combined potential; (a) (b) and (c) (d) for $\Omega/\omega_\perp$ = 2.5 and 3.2, respectively. The phase varies continuously from 0(black) to 2π(white). The discontinuous lines between black and white are the branch cuts whose edges represent vortices(phase defects).

inside the hole, although they are packed closely.

The ring structure of Fig. 2(c) opens the new experimental research of superfludity in this system. First, phase slippage [8] will be observed well under control. When Ω is decreased suddenly, some of the packed defects spiral out and go across the circular superflow, which results in a decrease in v_s; this is nothing but phase slippage, which is confirmed by the measurement of the angular momentum [9]. Second, the giant vortex sustains a quasi-one-dimensional superflow free from impurities and walls; supersonic flow beyond the Landau critical velocity can be realized [5].

References

[1] K.W.Madison *et al.*, Phys. Rev. Lett. **84** (2000) 806.

[2] J.R.Abo-Shaeer *et al.*, Science **292** (2001) 476; P.C.Haljan *et al.*, Phys. Rev. Lett. **87** (2001) 210403; E.Hodby *et al.*, Phys. Rev. Lett. **88** (2002) 010405.

[3] L.J.Campbell, R.M.Ziff, Phys. Rev. B **20** (1979) 1886.

[4] M.Tsubota, K.Kasamatsu, M.Ueda, Phys. Rev. A **65** (2002) 023603.

[5] K.Kasamatsu, M.Tsubota, M.Ueda, cond-mat/0202223.

[6] You can see the animation of this dynamics in http://matter.sci.osaka-cu.ac.jp/bsr/vortexex-e.html.

[7] A.L.Fetter, Phys. Rev. A **64**(2001) 063608; T.L. Ho, Phys. Rev. Lett. **87** (2001) 060403.

[8] P.W.Anderson, Rev. Mod. Phys. **38** (1966) 298.

[9] F.Chevy *et al.*, Phys. Rev. Lett. **85** (2000) 2223.