

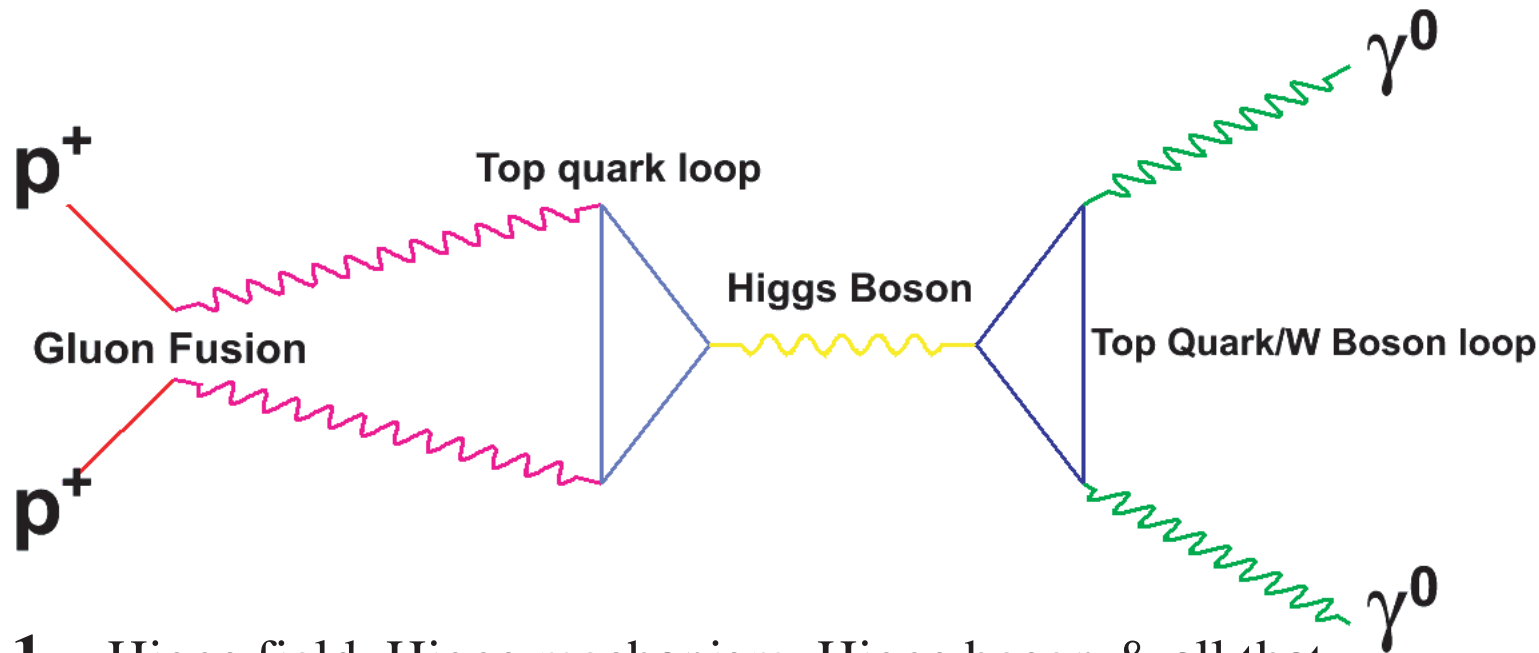
# Higgs bosons in particle physics & condensed matter

$\hat{a}^+ \hat{a}$  Aalto University

*G. Volovik*

Landau Institute

RUSSIAN ACADEMY OF SCIENCES  
L.D Landau  
INSTITUTE FOR  
THEORETICAL  
PHYSICS



1. Higgs field, Higgs mechanism, Higgs boson & all that
2.  ${}^3\text{He-B}$  & Nambu conjecture on masses of fermions & bosons
3. from  ${}^3\text{He-B}$  to Standard Model: application of Nambu sum rule
4. do we see extra Higgs bosons ?
5. Higgs bosons from quench (cosmology & superconductors)
6. Majorana & Majorino



in collaboration with Mikhail Zubkov, ITEP, Moscow

*Emergence & effective theories*  
*Weyl, Majorana & Dirac fermions*

*Vacuum polarization, screening - antiscreening, running coupling*  
*Symmetry breaking (anisotropy of vacuum)*  
*Parity violation -- chiral fermions*  
*Vacuum instability in strong fields, pair production*  
*Casimir force, quantum friction*  
*Fermionic charge of vacuum*

**Higgs fields, gauge bosons, Higgs bosons title 2013**

*Momentum-space topology*  
*Hierarchy problem, Supersymmetry*

*Torsion & spinning strings, torsion instanton*  
*Fermion zero modes on strings & walls*  
*Antigravitating (negative-mass) string*  
*Gravitational Aharonov-Bohm effect*  
*Domain wall terminating on string*  
*String terminating on domain wall*  
*Monopoles on string & Boojums*  
*Witten superconducting string*  
*Soft core string, Q-balls*

*Z & W strings*  
*Kibble mechanism*  
*Dark matter detector*  
*Primordial magnetic field*  
*Baryogenesis by textures*  
*Cosmological & Newton constants*  
*dark energy*  
*dark matter*  
*Effective gravity*  
*Bi-metric & conformal gravity*  
*Graviton, dilaton*  
*Spin connection*  
*Rotating vacuum*  
*Vacuum dynamics*  
*conformal anomaly*  
*ergoregion, event horizon*  
*Hawking & Unruh effects*  
*black hole entropy*  
*Vacuum instability*  
*Superfluidity of neutron star*  
*vortices, glitches*  
*shear flow instability*

*& strings*  
*Inflation*  
*Branes*  
*matter creation*  
*vacuum gravity*  
*black holes*  
*neutron stars*  
*quark matter*  
*nuclear physics*  
*phase transitions*  
*disorder*  
*random anisotropy*  
*Larkin-Imry-Ma classes of random matrices*  
*Color superfluidity*  
*Savvidi vacuum*  
*Quark confinement, QCD cosmology*  
*Intrinsic orbital momentum of quark matter*

cosmic strings

physical vacuum

cosmology

High Energy Physics

high-T & chiral superconductivity

vacuum gravity

Gravity

3He

Condensed Matter

1D, 2D systems

topological insulators, semimetals

black holes

QCD

Plasma Physics

Phenomenology

BEC

neutron stars

quark matter

nuclear physics

phase transitions

hydrodynamics

*Quark condensate*  
*Nambu--Jona-Lasinio*  
*Vaks--Larkin*  
*Color superfluidity*  
*Savvidi vacuum*  
*Quark confinement, QCD cosmology*  
*Intrinsic orbital momentum of quark matter*

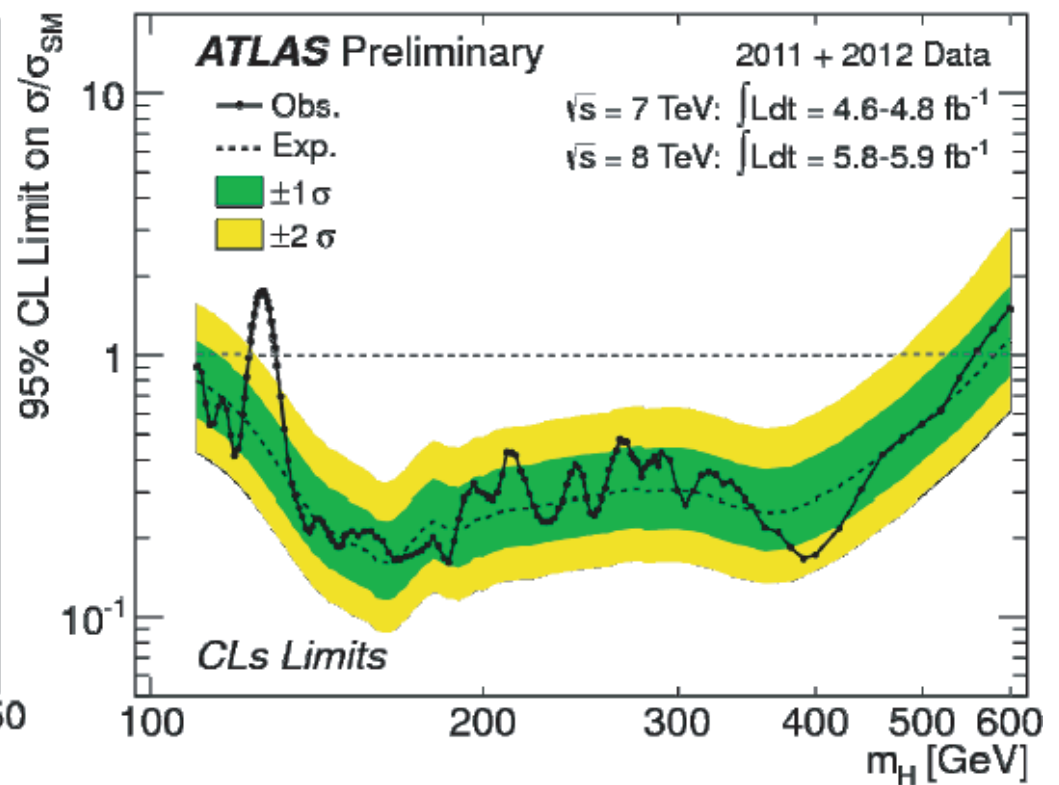
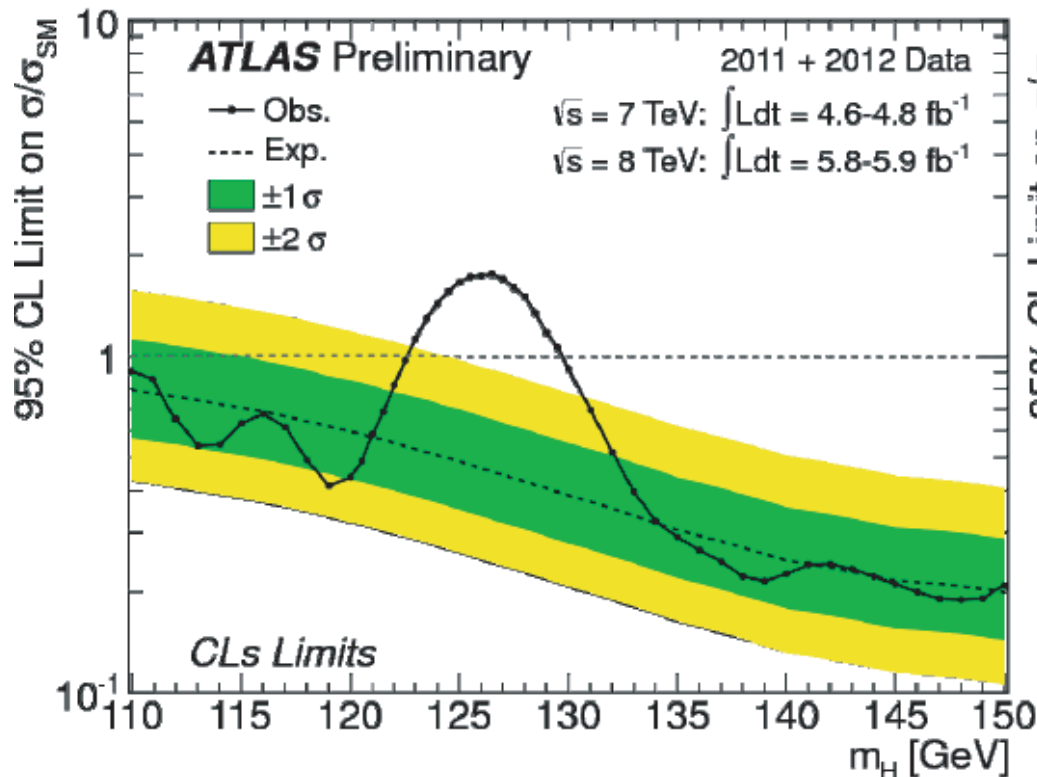
*Nuclei vs 3He droplet*  
*Shell model*  
*Pair-correlations*  
*Collective modes*  
*QCD cosmology*

*quantum phase transitions & momentum-space topology*  
*Relativistic plasma*  
*Photon mass*  
*Vortex Coulomb plasma*

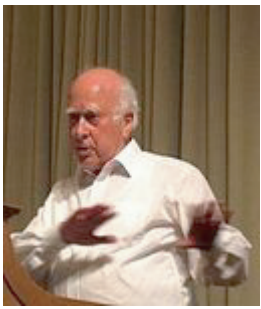
*random anisotropy*  
*Larkin-Imry-Ma classes of random matrices*

*Gap nodes*  
*Low -T scaling*  
*mixed state*  
*Broken time reversal*  
*1/2-vortex, vortex dynamics*  
*Films: FQHE, Statistics & charge of skyrmions & vortices*  
*Edge states; flat band, Fermi arc spintronics*  
*1D fermions in vortex core*  
*Critical fluctuations*  
*Mixture of condensates*  
*Vector & spinor condensates*  
*BEC of quasiparticles, magnon BEC & laser*  
*meron, skyrmion, 1/2 vortex*  
*General; relativistic; spin superfluidity*  
*multi-fluid*  
*rotating superfluid*  
*Shear flow instability*  
*Magnetohydrodynamic*  
*Turbulence of vortex lines*  
*propagating vortex front*  
*velocity independent*  
*Reynolds number*

- Combining updated 2011+2012  $\gamma\gamma$  and 4-lepton analysis with others as before



- Expected exclusion with this dataset from 110-582 GeV
  - Observed exclusion from 110-122.6 GeV and 129.7-558 GeV
    - 111.7-121.8 GeV and 130.7-523 GeV excluded at 99% CL
  - Region around 126 GeV cannot be excluded



**author of Higgs mechanism & massive Higgs bosons**

**co-authors of Higgs mechanism**



**Kibble**

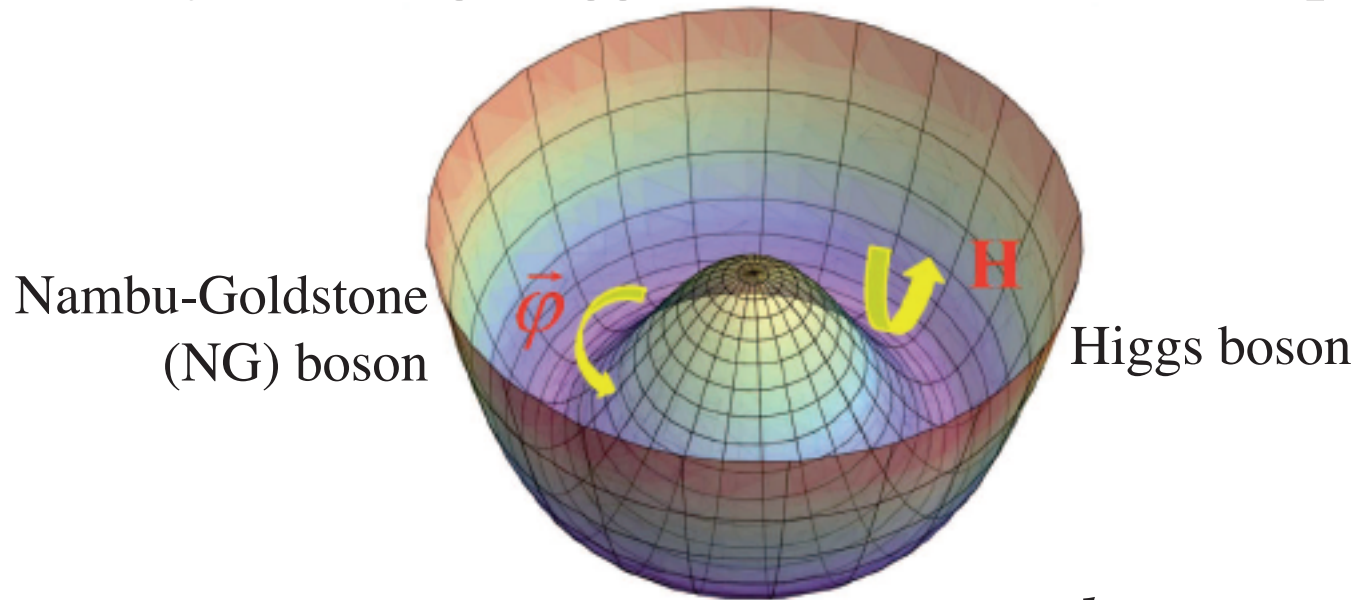
**Guralnik**

**Hagen**

**Englert**

**Brout**

# Symmetry breaking, Higgs mechanism & massive particles



*Standard Model*

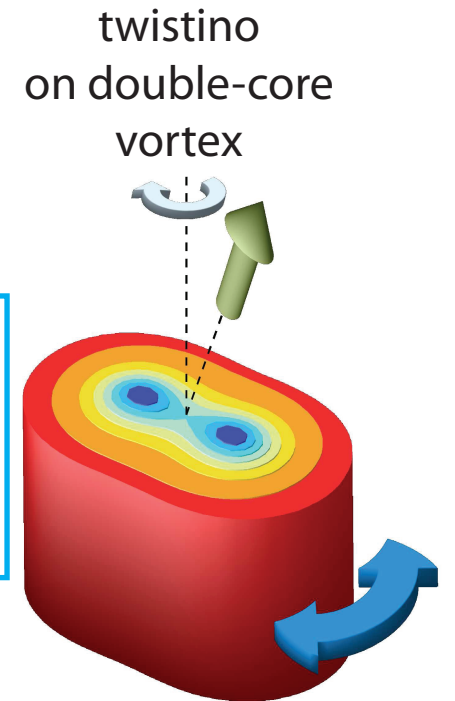
*cond-mat*

NG bosons

no NG bosons !

phonon, spin wave, orbital wave, Kelvin (Kelvin wave), twistino, Tkachenko wave, ripplon, Parshin (crystallization) wave, ...

massive particles



Higgs boson(s)

order parameter modes

W & Z gauge bosons

Meissner effect (expulsion of magnetic field)

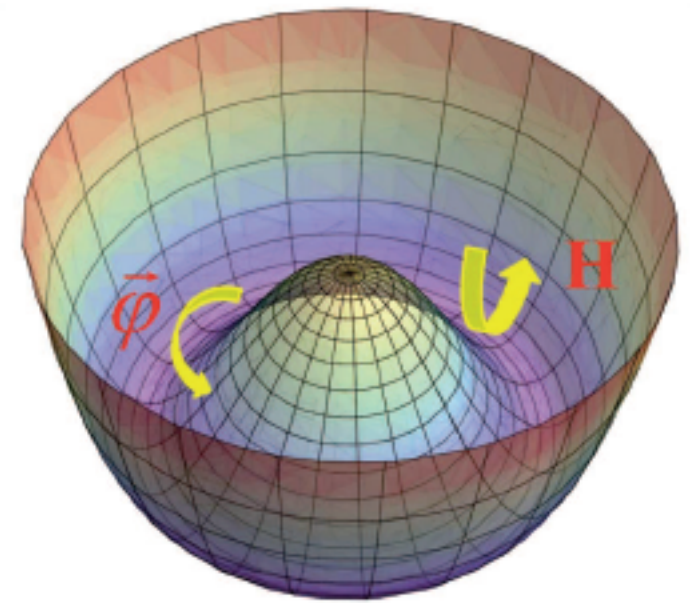
quarks & leptons

gapped quasiparticles

# Nambu-Goldstone (NG) bosons

Goldstone's theorem:

spontaneous breakdown of a global continuous symmetry leads to massless particle – Nambu-Goldstone boson



Y. Nambu, Phys. Rev. Lett. **4** (1960) 380

J. Goldstone, Nuovo Cim. **19** (1961) 154

J. Goldstone, A. Salam, S. Weinberg, Phys. Rev. **127** (1962) 965

cond-mat order parameter = Higgs field

$$\Psi = |\Psi| e^{i\varphi}$$

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

cond-mat NG bosons = gapless modes of order parameter:  
phonons, spin waves, orbital waves

no NG bosons had been observed in particle physics

**Why ?**

# Higgs mechanism

spontaneous breaking of gauge symmetry does not require Goldstone bosons, their degrees of freedom deliver the longitudinal polarization modes of gauge bosons, gauge bosons become massive (Z-boson, W-boson)

P. Anderson, Plasmons, gauge invariance and mass, Phys. Rev. **130** (1963) 439

F. Englert, R. Brout, Phys. Rev. Lett. **13** (1964) 321

P. W. Higgs, Phys. Lett. **12** (1964) 132 and Phys. Rev. Lett. **13** (1964) 508

G. S. Guralnik, C. R. Hagen, T. W. B. Kibble, Phys. Rev. Lett. **13** (1964) 585

cond-mat analog: Cooper pairing and Meissner effect in superconductors

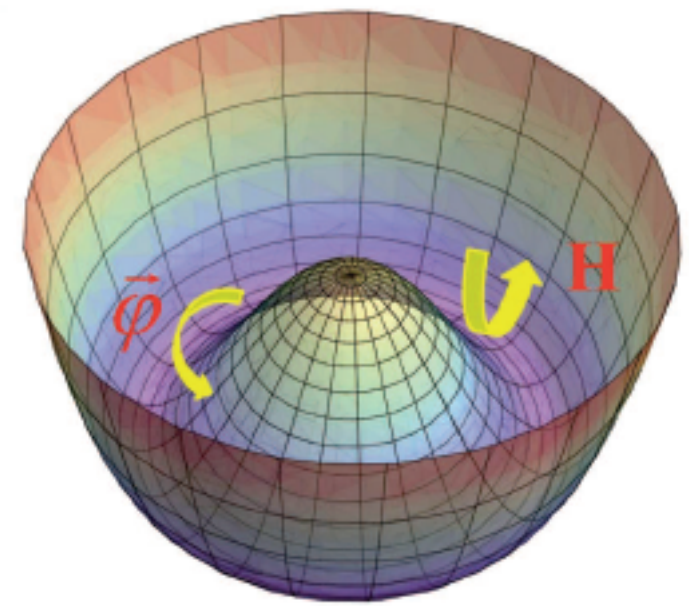
Higgs, PRL **13** (1964) 508:

“It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.”

“In the theory of superconductivity the scalar fields are associated with fermion pairs; the doubly charged excitation responsible for the quantization of magnetic flux is then the surviving member of a U(1) doublet.”

# How many Higgs bosons ?

amplitude H-modes of Higgs field



P. W. Higgs, PRL **13** (1964) 508:

“The model of the most immediate interest is that in which the scalar fields form an **octet** under **SU(3)**... There are **2** massive scalar bosons ... (**2 Higgs bosons**) ... ; the remaining **6** components of the scalar octet combine with the corresponding components of the gauge-field octet to describe massive vector bosons (**6 massive gauge bosons**).”

Vdovin (1963)

**Bosons in superfluid  $^3\text{He-B}$ :** collective modes of order parameter

**9-plet  $A_{\alpha i}$**  under **SO(3) x SO(3) x U(1)**

**14 Higgs bosons + 4 NG modes** (**4 massive gauge bosons** in gauged  $^3\text{He-B}$ )



# Masses of elementary particles

elementary particles acquire mass due to interaction with Higgs field

cond-mat mass of elementary particles:

Bogoliubov- de Gennes particles with gap in superconductors and  $^3\text{He-B}$

# Ginzburg-Landau

vs

# Standard Model

Order parameter:  
complex scalar field

$$\Psi = |\Psi| e^{i\varphi}$$

2 = 1+1 collective modes:

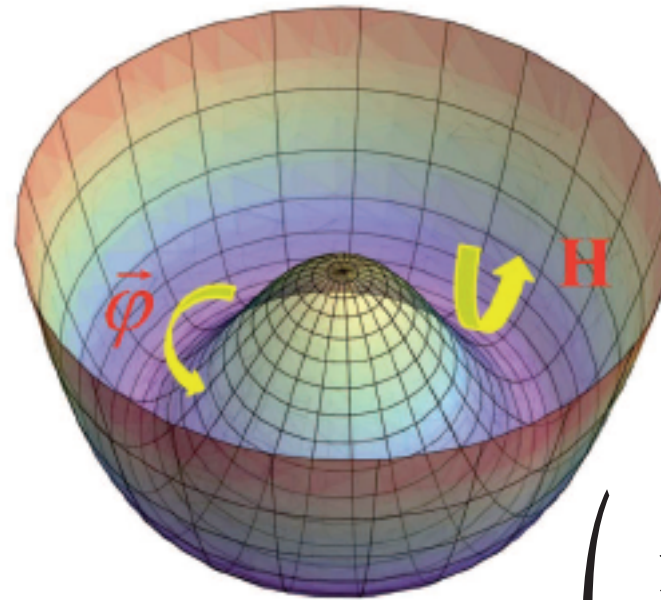
$$\Psi = 1 + (H + i\varphi)$$

1 NG mode (phase  $\varphi$  mode)

1 amplitude  $H$  mode (Higgs boson)

Meissner effect:

mass of gauge vector boson =  
inverse penetration length



Higgs field: complex spinor

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

4 = 3+1 collective modes

$$\Psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + H \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \delta\Psi$$

3 rotations

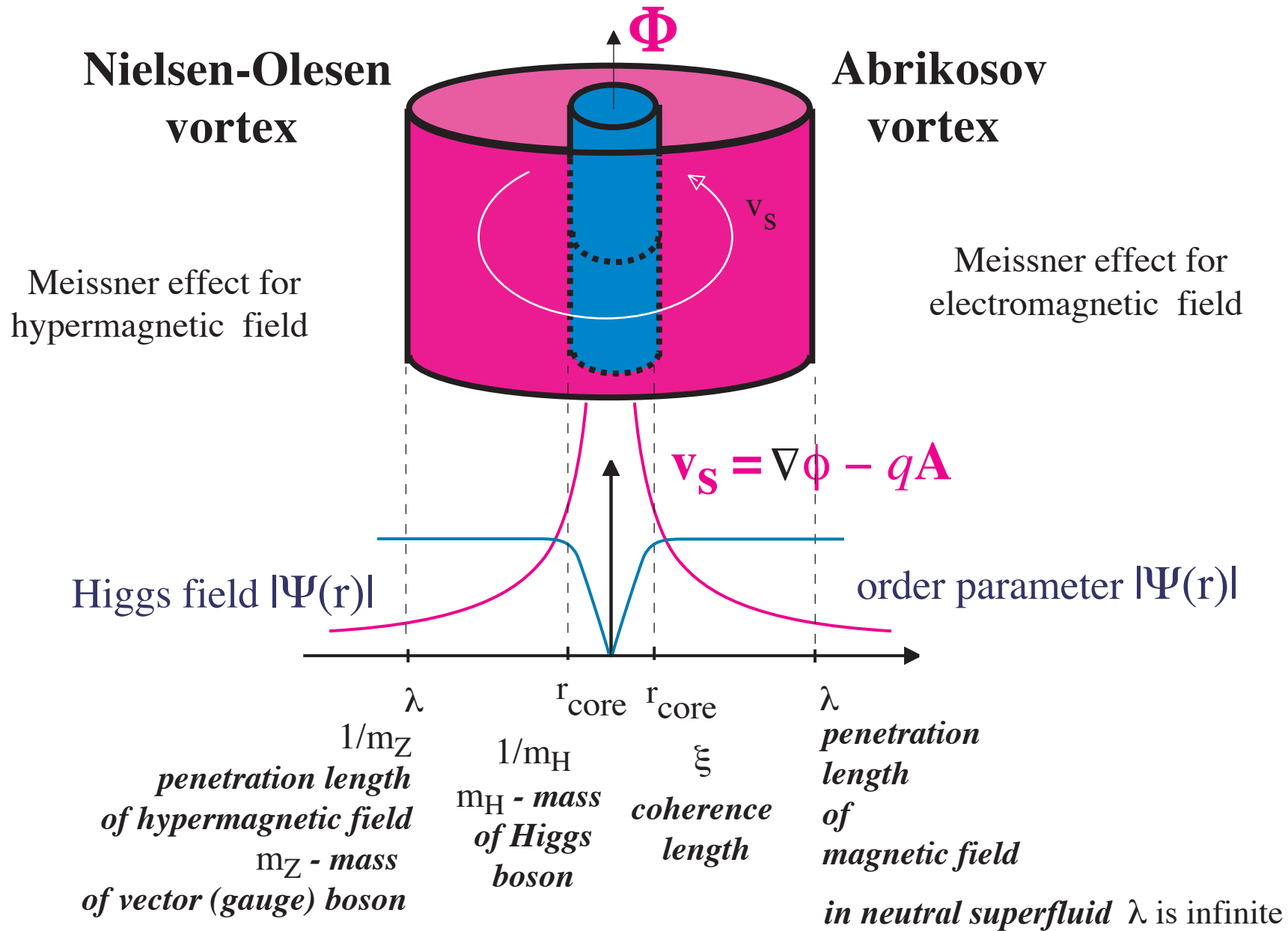
3 NG modes

1 amplitude  $H$  mode (Higgs boson  $H$ )

**Standard Model requires only one Higgs boson**

3 NG modes are absorbed into longitudinal  
modes of 3 massive vector bosons  
(charged  $W^+$ ,  $W^-$  and neutral  $Z$ )  
this is called **Higgs mechanism**

# Higgs mechanism in Standard Model & superconductor



$$E = |\nabla\Psi - iq\mathbf{A}\Psi|^2 = |\Psi|^2(\nabla\phi - q\mathbf{A})^2 = \lambda^{-2}(\nabla\phi - q\mathbf{A})^2 = \rho_s v_s^2$$

# from Standard Model Ginzburg-Landau to Standard Model BEC

nobody now believes that SM is complete theory,  
all believe SM is effective low energy theory

**composite Higgs is more natural than fundamental**

Higgs, PRL **13** (1964) 508:

“... the symmetry-breaking scalar fields  
are not elementary dynamic variables  
but *bilinear combinations of Fermi fields.*”

Higgs field can be composite object as Cooper pair in superconductors

$$\Psi = \langle \bar{\chi}_L \chi_R \rangle$$

# Standard Model as BCS theory in semimetal

BCS Cooper pairing

dynamical mixing of left & right particles

Order parameter as composite object

Higgs field as composite object

$$\Psi = \langle \chi \chi \rangle$$

$$\Psi = \langle \bar{\chi}_L \chi_R \rangle$$

phase & amplitude modes

Higgs bosons

$$\Psi = \langle \chi \chi \rangle_{\text{vac}} + \delta\Psi$$

$$\Psi = \langle \bar{\chi}_L \chi_R \rangle_{\text{vac}} + \delta\Psi$$

$\Psi$  mixes electron and hole

$\Psi$  mixes left and right Weyl particles

Hamiltonian for Bogoliubov-Nambu quasiparticles

Hamiltonian for Dirac particles

$$H = \begin{pmatrix} \varepsilon(\mathbf{p}) & \Psi \\ \Psi^* & -\varepsilon(\mathbf{p}) \end{pmatrix}$$

$$H = \begin{pmatrix} c\boldsymbol{\sigma}\cdot\mathbf{p} & \Psi \\ \Psi^+ & -c\boldsymbol{\sigma}\cdot\mathbf{p} \end{pmatrix}$$

order parameter induces gap

Higgs field induces Dirac mass  $M = |\Psi|$

$$E^2(\mathbf{p}) = \varepsilon^2(\mathbf{p}) + |\Psi|^2$$

$$E^2(\mathbf{p}) = c^2 p^2 + |\Psi|^2$$

# generalization of Standard Model using hints from superfluid $^3\text{He}$

superfluid  $^3\text{He}$

$$\text{order parameter } A_{\alpha j} = \langle \chi \sigma_{\alpha} \nabla_j \chi \rangle = \langle \chi \hat{O} \chi \rangle$$

in superfluid  $^3\text{He}$

$$\hat{O} = \sigma_{\alpha} \nabla_j$$

$\hat{O}$  is in general operator, which may include generators of different symmetry groups

general composite Higgs field

$$\text{Higgs field } \Psi = \langle \bar{\chi} \hat{O} \chi \rangle$$

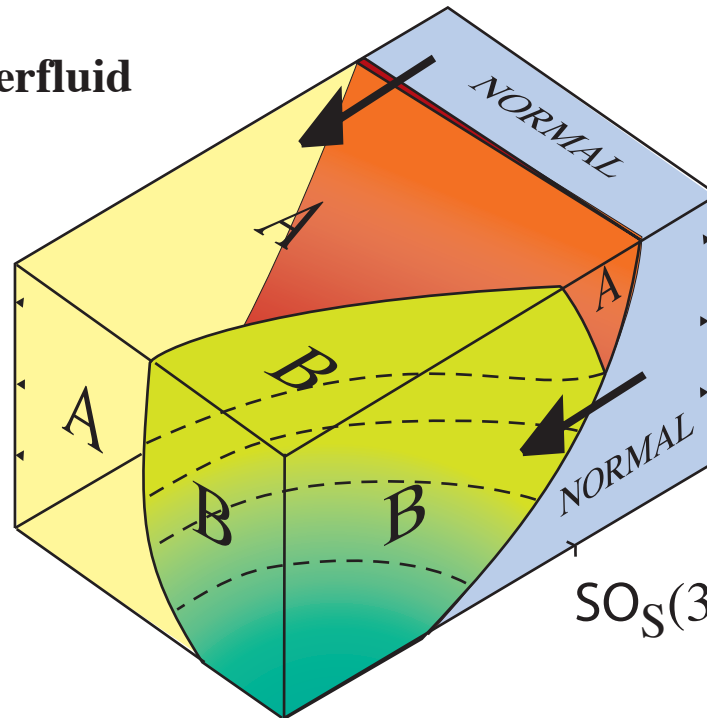
*GUT in Standard Model  
symmetry breaking phase transitions*

$$SO(10) \rightarrow SU(3) \times SU_L(2) \times U(1) \rightarrow SU(3) \times U_Q(1)$$

*symmetry breaking phase transitions  
in superfluid  $^3\text{He}$*

$$SO_S(3) \times SO_L(3) \times U(1) \rightarrow SO_S(2) \times U_Q(1) \quad Q \text{ is analog of electric charge}$$

$^3\text{He-A}$   
topological chiral Weyl superfluid  
two Dirac cones



$^3\text{He-B}$   
time-reversal symmetric  
topological superfluid

$$SO_S(3) \times SO_L(3) \times U(1) \rightarrow SO_J(3)$$

# multiple Higgs bosons from superfluid $^3\text{He}$

superfluid  $^3\text{He}$

order parameter  $A_{\alpha j} = \langle \chi \sigma_{\alpha} \nabla_j \chi \rangle$

pair breaking & squashing modes

$$A_{\alpha j} = \delta_{\alpha j} + u_{\alpha j} + i v_{\alpha j}$$

$\hat{O}$  is operator, which may include generators of different symmetry groups

Bogoliubov-Nambu quasiparticles

composite Higgs

Higgs field  $\Psi = \langle \bar{\chi} \hat{O} \chi \rangle$

multiple Higgs bosons

$$\Psi = \langle \bar{\chi} \hat{O} \chi \rangle_{\text{vac}} + \delta\Psi$$

massive quarks & leptons

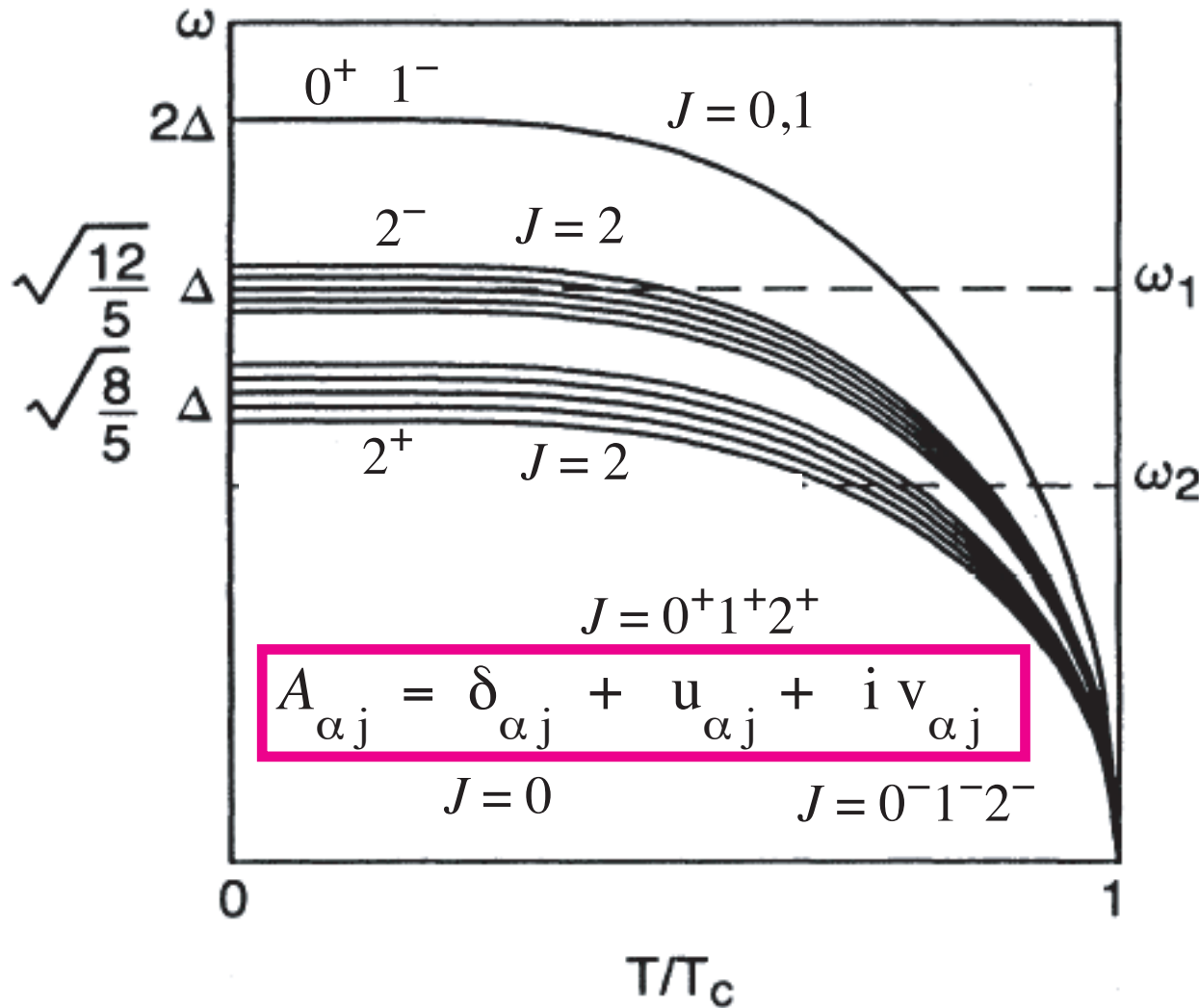
$$H = \begin{pmatrix} \varepsilon(\mathbf{p}) & A_{\alpha j} \sigma_{\alpha} \cdot p_j \\ A_{\alpha j}^* \sigma_{\alpha} \cdot p_j & -\varepsilon(\mathbf{p}) \end{pmatrix}$$

$$H = \begin{pmatrix} c\sigma \cdot \mathbf{p} & & & \\ & c\sigma \cdot \mathbf{p} & & \Psi \\ & & -c\sigma \cdot \mathbf{p} & \\ \Psi^+ & & & -c\sigma \cdot \mathbf{p} \end{pmatrix}$$



# 4 Goldstone & 14 Higgs modes in $^3\text{He-B}$

$$\text{SO}_S(3) \times \text{SO}_L(3) \times \text{U}(1) \longrightarrow \text{SO}_J(3)$$



## 4 Goldstone modes

$$E_{0-} = E_{1+} = 0$$

## 14 Higgs bosons

$$E_{1-} = E_{0+} = 2\Delta$$

$$E_{2-} = (12/5)^{1/2}\Delta$$

$$E_{2+} = (8/5)^{1/2}\Delta$$

## 9 pairs of Nambu partners

$$E_{2-}^2 + E_{2+}^2 = 4 \Delta^2$$

$$E_{0-} + E_{0+} = 4 \Delta^2$$

$$E_{1-} + E_{1+} = 4 \Delta^2$$

FIG. 3. A schematic plot in  $\omega$  vs  $T$  space for pair breaking, the squashing mode, and the real squashing modes. The Zeeman splitting (not to scale) of the collective modes in an applied magnetic field is shown in the plot. The dashed lines labeled  $\omega_1$  and  $\omega_2$  correspond to two sound frequencies.

## Nambu sum rule: from 3He-B to Standard Model

relation between energies  $E_B$  &  $E_F$  of bosonic & fermionic excitations in BCS type theories

$$\boxed{E_{B1}^2} + \boxed{E_{B2}^2} = \boxed{4 E_F^2}$$

3He-B	$E_{B1} = (8/5)^{1/2} \Delta$	$E_{B2} = (12/5)^{1/2} \Delta$	$E_F = \Delta$
	real squashing mode	squashing mode	gap in quasiparticle spectrum

Application of Nambu rule to masses of Higgs fields and top quarks

$$\boxed{m_{H1}^2} + \boxed{m_{H2}^2} = \boxed{4 m_t^2}$$

125 GeV

325 GeV

174 GeV

discovered Higgs

Nambu partner Higgs

top quark

# Sum rule for $^3\text{He-B}$ : symmetry consideration

$$m_{\text{Q}}^2 = 2m_{\text{f}}^2 (1 \pm \eta^{(\text{Q})})$$

parameter dictated by symmetry

$$\eta^{(\text{Q})} = \text{Tr} (\text{VO}^{(\text{Q})}\text{VO}^{(\text{Q})})$$

$$m_{\text{Q}^+}^2 + m_{\text{Q}^-}^2 = 4 m_{\text{f}}^2$$

$^3\text{He-B}$ : quantum number  $\text{Q} = \text{J}$  (total angular momentum),  $m_{\text{f}} = \Delta$  gap

$$E_{\text{J}}^2 = 2 \Delta^2 (1 \pm \eta^{(\text{J})})$$

$$\text{J}=2, \eta^{(\text{J})} = 1/5$$

$$E_{2-} = (8/5)^{1/2} \Delta$$

real squashing mode

$$E_{2+} = (12/5)^{1/2} \Delta$$

squashing mode

$$\text{J}=0, \eta^{(\text{J})} = 1$$

$$E_{1-} = 0$$

Goldstone  
(sound mode)

$$E_{1+} = 2 \Delta$$

pair breaking mode

$$\text{J}=1, \eta^{(\text{J})} = 1$$

$$E_{0-} = 0$$

3 Goldstone modes  
(spin waves)

$$E_{0+} = 2 \Delta$$

pair breaking modes

# which fermion is responsible?

# Standard Model

Three generations  
of matter (fermions)

	I	II	III	
mass →	2.4 MeV/c <sup>2</sup>	1.27 GeV/c <sup>2</sup>	171.2 GeV/c <sup>2</sup>	0
charge →	<sup>2</sup> / <sub>3</sub>	<sup>2</sup> / <sub>3</sub>	<sup>2</sup> / <sub>3</sub>	0
spin →	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	1
name →	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>γ</b> photon
Quarks	4.8 MeV/c <sup>2</sup>	104 MeV/c <sup>2</sup>	4.2 GeV/c <sup>2</sup>	0
	<sup>-1</sup> / <sub>3</sub>	<sup>-1</sup> / <sub>3</sub>	<sup>-1</sup> / <sub>3</sub>	0
	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	1
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b>g</b> gluon
Leptons	<2.2 eV/c <sup>2</sup>	<0.17 MeV/c <sup>2</sup>	<15.5 MeV/c <sup>2</sup>	91.2 GeV/c <sup>2</sup>
	0	0	0	0
	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	1
	<b>ν<sub>e</sub></b> electron neutrino	<b>ν<sub>μ</sub></b> muon neutrino	<b>ν<sub>τ</sub></b> tau neutrino	<b>Z<sup>0</sup></b> Z boson
	0.511 MeV/c <sup>2</sup>	105.7 MeV/c <sup>2</sup>	1.777 GeV/c <sup>2</sup>	80.4 GeV/c <sup>2</sup>
	-1	-1	-1	±1
	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	1
	<b>e</b> electron	<b>μ</b> muon	<b>τ</b> tau	<b>W<sup>±</sup></b> W boson

if pairing occurs in one channel  
fermion in this channel  
will have largest gap (mass)

Gauge bosons

## Nambu sum rule: from 3He-B to Standard Model

relation between energies  $E_B$  &  $E_F$  of bosonic & fermionic excitations in BCS type theories

$$\boxed{E_{B1}^2} + \boxed{E_{B2}^2} = \boxed{4 E_F^2}$$

3He-B	$E_{B1} = (8/5)^{1/2} \Delta$	$E_{B2} = (12/5)^{1/2} \Delta$	$E_F = \Delta$
	real squashing mode	squashing mode	gap in quasiparticle spectrum

Application of Nambu rule to masses of Higgs fields and top quarks

$$\boxed{m_{H1}^2} + \boxed{m_{H2}^2} = \boxed{4 m_t^2}$$

125 GeV

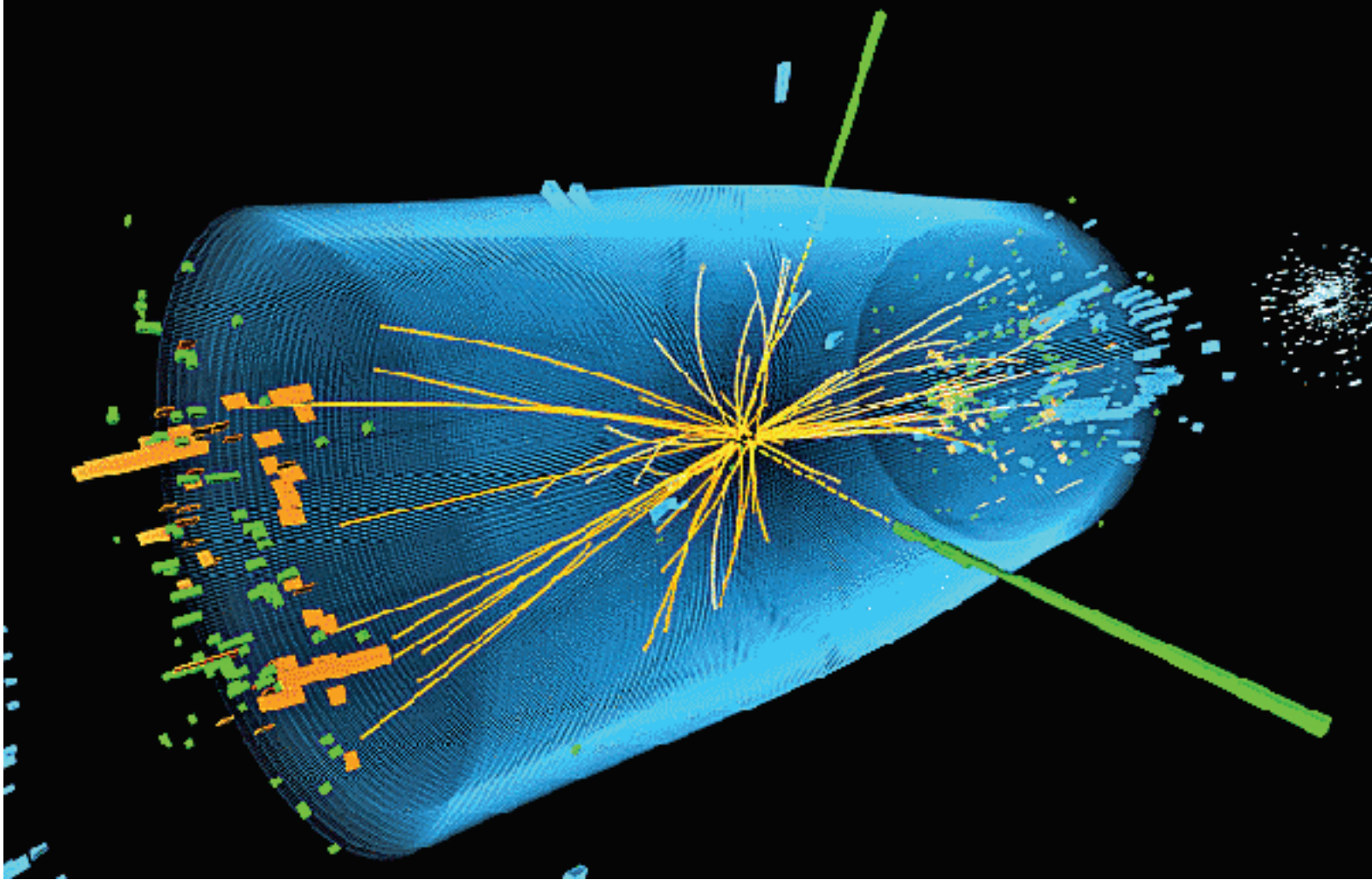
325 GeV

174 GeV

discovered Higgs

Nambu partner Higgs

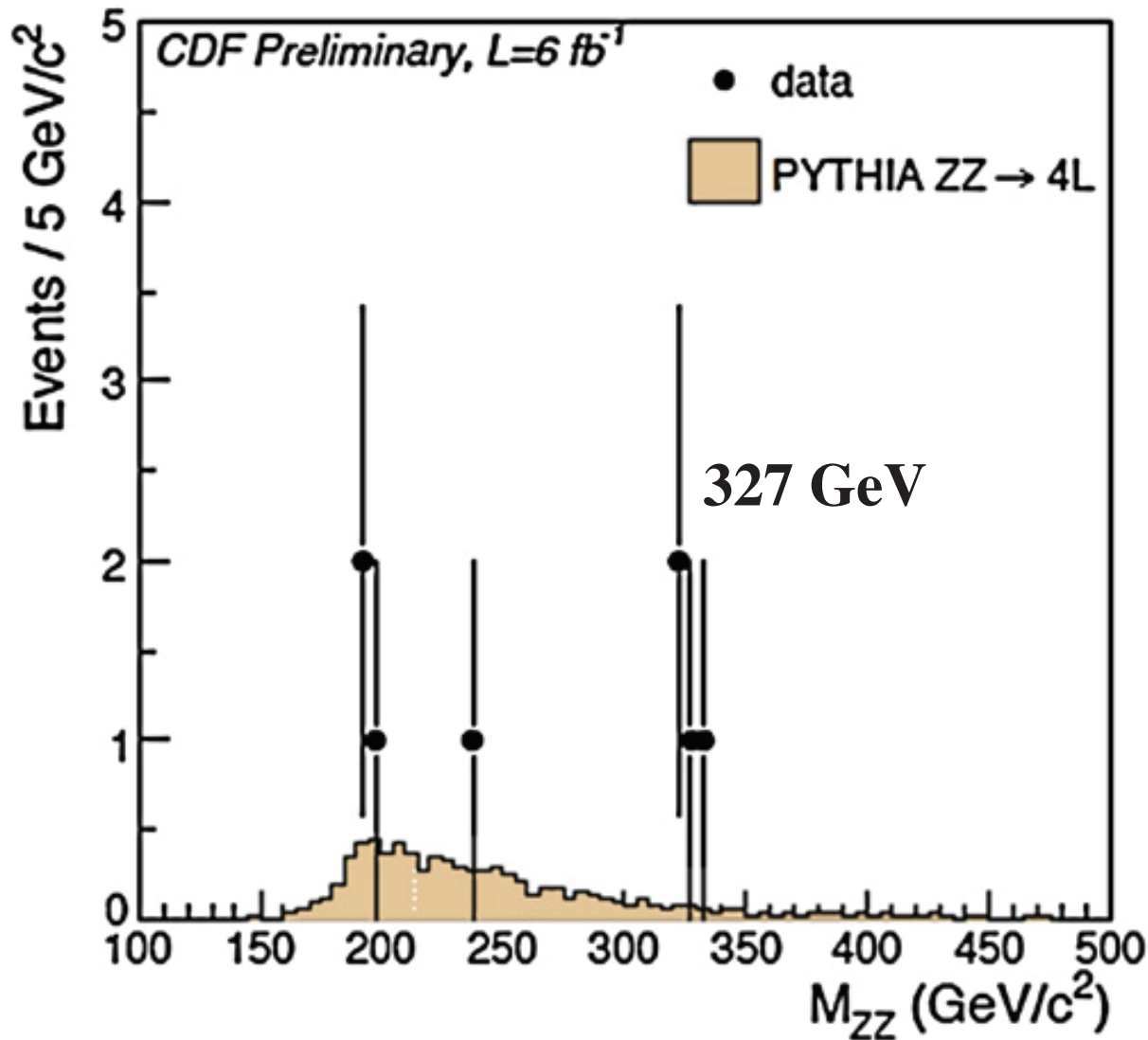
top quark



# A narrow scalar resonance at 325 GeV?

Krzysztof A. Meissner & Hermann Nicolai

Phys. Lett. B **718** (2013) 943



“We propose to identify the excess of events with four charged leptons at  $E \sim 325$  GeV seen by the CDF Collaboration (2012) [1] & CMS Collaboration (2012) [2] with a new ‘sterile’ scalar particle characterized by a very narrow resonance of the same height and branching ratios as the Standard Model Higgs boson, as predicted in the framework of the so-called Conformal Standard Model (K.A. Meissner & H. Nicolai 2007) [3].”

Fig. 1. The four lepton events reported by the CDF Collaboration [1].

# hints of 325 GeV Higgs boson in earlier experiments

\* CDF/PUB/EXOTICS/PUBLIC/10603 July 17, 2011  
Search for High-Mass Resonances Decaying  
into ZZ in pp Collisions at  $\sqrt{s} = 1.96\text{TeV}$

"The invariant masses of four events  
are clustered around 325 GeV"

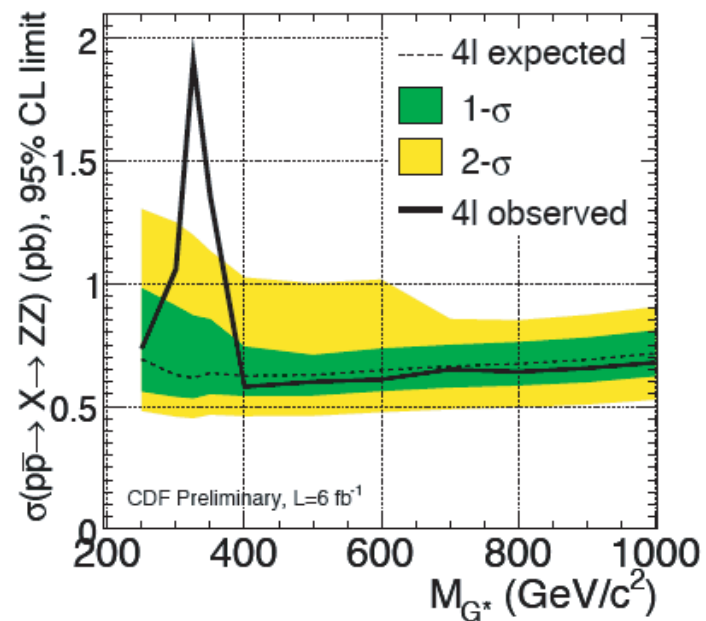
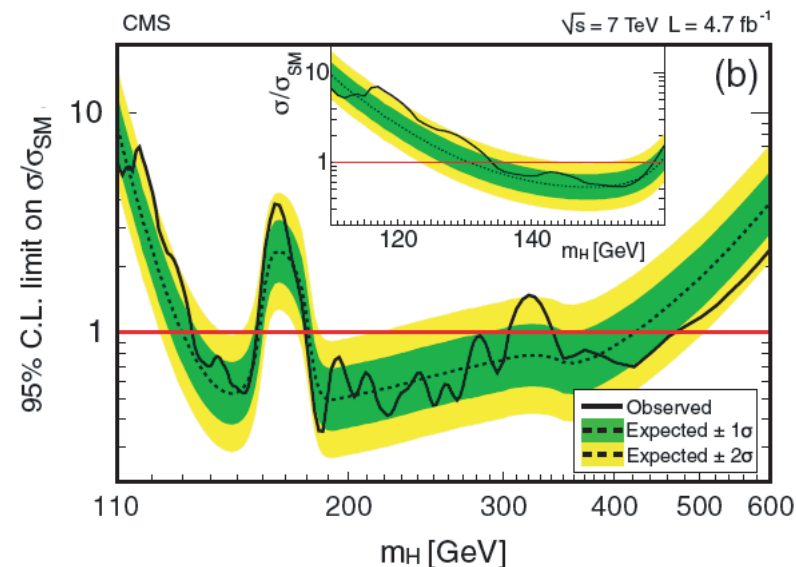


FIG. 13: Expected and observed 95% CL limits on  $\sigma(pp \rightarrow X \rightarrow ZZ)$  from the  $ZZ \rightarrow \ell^+ \ell^- \ell^+ \ell^-$  channel; the four events with  $M_{ZZ} = 327 \text{ GeV}/c^2$  result in a deviation from the expected limit.

\* Search for the Standard Model Higgs Boson  
in CMS Collaboration @ LHC  
(Compact Muon Solenoid)

PRL 108, 111804 (2012)

"Small excesses of events are observed around  
masses of 119, 126 & 320 GeV"





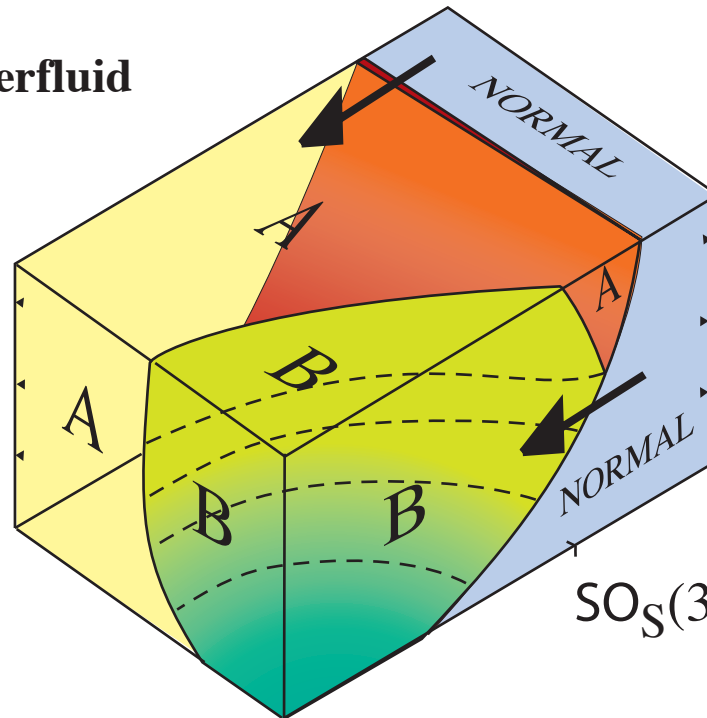
*GUT in Standard Model  
symmetry breaking phase transitions*

$$SO(10) \rightarrow SU(3) \times SU_L(2) \times U(1) \rightarrow SU(3) \times U_Q(1)$$

*symmetry breaking phase transitions  
in superfluid  $^3\text{He}$*

$$SO_S(3) \times SO_L(3) \times U(1) \rightarrow SO_S(2) \times U_Q(1) \quad Q \text{ is analog of electric charge}$$

$^3\text{He-A}$   
topological chiral Weyl superfluid  
two Dirac cones



$^3\text{He-B}$   
time-reversal symmetric  
topological superfluid

$$SO_S(3) \times SO_L(3) \times U(1) \rightarrow SO_J(3)$$

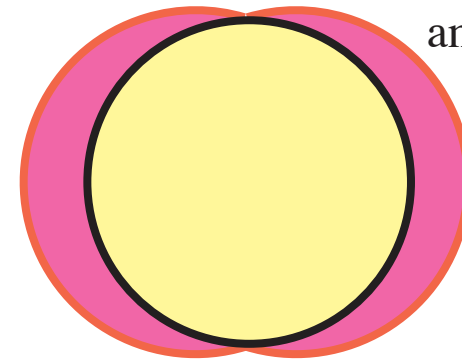
# 9 Goldstone + 9 Higgs modes in bulk ${}^3\text{He-A}$

$U_Q(1)$  - analog of electromagnetic group

quantum number  $Q$  - analog of electric charge

$$SO_5(3) \times SO_L(3) \times U(1) \longrightarrow SO_5(2) \times U_Q(1)$$

Modes	Variables	In the absence of dipole interaction and magnetic field			
		$Q$	$p^z$	$S_z$	$p^x$
Sound $E = 0$	$u_{23} - v_{13}$	0	-1	0	+1
Spin waves $E = 0$	$u_{11} + v_{21},$	0	+1	$\pm 1$	-
	$u_{12} + v_{22}$				
Orbital modes	$u_{33}, v_{33}$	$\pm 1$	-	0	-1
Spin-orbit modes $E = 0$	$u_{31}, v_{31}$				
	$u_{32}, v_{32}$	$\pm 1$	-	$\pm 1$	-
$E^2 = 8/3 \Delta_0^2$	$u_{13} + v_{23}$	0	+1	0	+1
	$u_{22} - v_{12},$ $v_{11} - u_{21}$	0	-1	$\pm 1$	-
$E^2 = 4/3 \Delta_0^2$	$u_{25} + v_{13},$ $u_{13} - v_{23}$	$\pm 2$	-	0	+1
	"Clapping" modes $u_{11} - v_{21},$ $u_{21} + v_{11},$ $u_{12} - v_{22},$ $u_{22} + v_{12}$	$\pm 2$	-	$\pm 1$	-



anisotropic gap with nodes

$$\Delta(\theta) = \Delta_0 \sin \theta$$

$$\langle \Delta^2 \rangle = 2/3 \Delta_0^2$$

3 Goldstones + 9 Higgs form 6 Nambu pairs

$$E_{0-}^2 + E_{0+}^2 = 4 \langle \Delta^2 \rangle$$

$$E_{0-}^2 + E_{0+}^2 = 4 \langle \Delta^2 \rangle$$

$$E_{-2}^2 + E_{+2}^2 = 4 \langle \Delta^2 \rangle$$

6 Goldstones

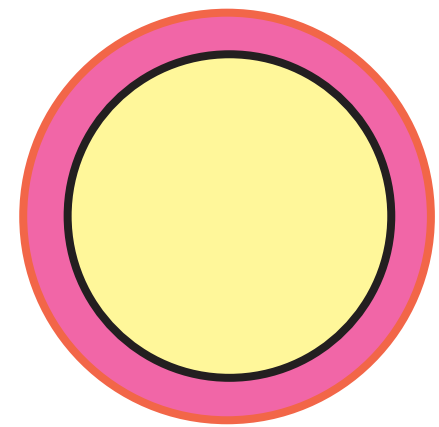
(2 "photons" + 4 from hidden symmetry)

have no Nambu partners

### 3 Goldstone + 9 Higgs modes in $^3\text{He-A}$ film

$$SO_5(3) \times SO_L(2) \times U(1) \longrightarrow SO_5(2) \times U_Q(1)$$

$$m_{Q+}^2 + m_{Q-}^2 = 4 m_f^2$$



isotropic gap  $\Delta$  in 2D

$^3\text{He-A}$  film: quantum number  $Q$  is analog of electric charge,  $m_f = \Delta$  gap

$$Q=0$$

$$E_{0-} = 0$$

Goldstone  
(sound mode)

$$E_{0+} = 2 \Delta$$

pair breaking mode

$$E_{0+}^2 + E_{0-}^2 = 4 \Delta^2$$

$$|Q|=1$$

$$E_{-1} = 2^{1/2} \Delta$$

$$E_{+1} = 2^{1/2} \Delta$$

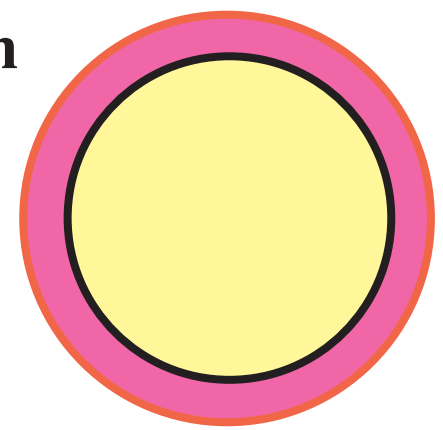
clapping modes  
analog of charged Higgs bosons

$$E_{-2}^2 + E_{+2}^2 = 4 \Delta^2$$

# Nambu sum rule for $^3\text{He-A}$ film: symmetry consideration

$$m_Q^2 = 2m_f^2 (1 \pm \eta^{(Q)})$$

$$\eta^{(Q)} = \text{Tr} (V O^{(Q)} V O^{(Q)})$$



isotropic gap  $\Delta$  in 2D

$^3\text{He-A}$  film: quantum number  $Q$  is analog of electric charge,  $m_f = \Delta$  gap

$$E_Q^2 = 2 \Delta^2 (1 \pm \eta^{(Q)})$$

$$Q=0, \eta^{(Q)} = 1$$

$$E_{0-} = 0$$

Goldstone  
(sound mode)

$$E_{0+} = 2 \Delta$$

pair breaking mode

$$E_{0+}^2 + E_{0-}^2 = 4 \Delta^2$$

$$Q=1, \eta^{(Q)} = 0$$

$$E_{-1} = 2^{1/2} \Delta$$

$$E_{+1} = 2^{1/2} \Delta$$

clapping modes  
analog of charged Higgs bosons

$$E_{-2}^2 + E_{+2}^2 = 4 \Delta^2$$

# Nambu sum rule: from ${}^3\text{He-A}$ to Standard Model

relation between energies  $E_B$  &  $E_F$  of bosonic & fermionic excitations  
in BCS type theories

$$E_{B1}^2 + E_{B2}^2 = 4 E_F^2$$

2D ${}^3\text{He-A}$	$E_{B+} = 2^{1/2}\Delta$	$E_{B-} = 2^{1/2}\Delta$	$E_F = \Delta$
	clapping modes		gap in quasiparticle spectrum

Application of Nambu rule to masses of charged Higgs fields and top quarks

$m_{H^+}^2$	$m_{H^-}^2$	$= 4 m_t^2$
245 GeV	245 GeV	174 GeV
charged Higgs	charged Higgs	top quark

experimental evidence ?

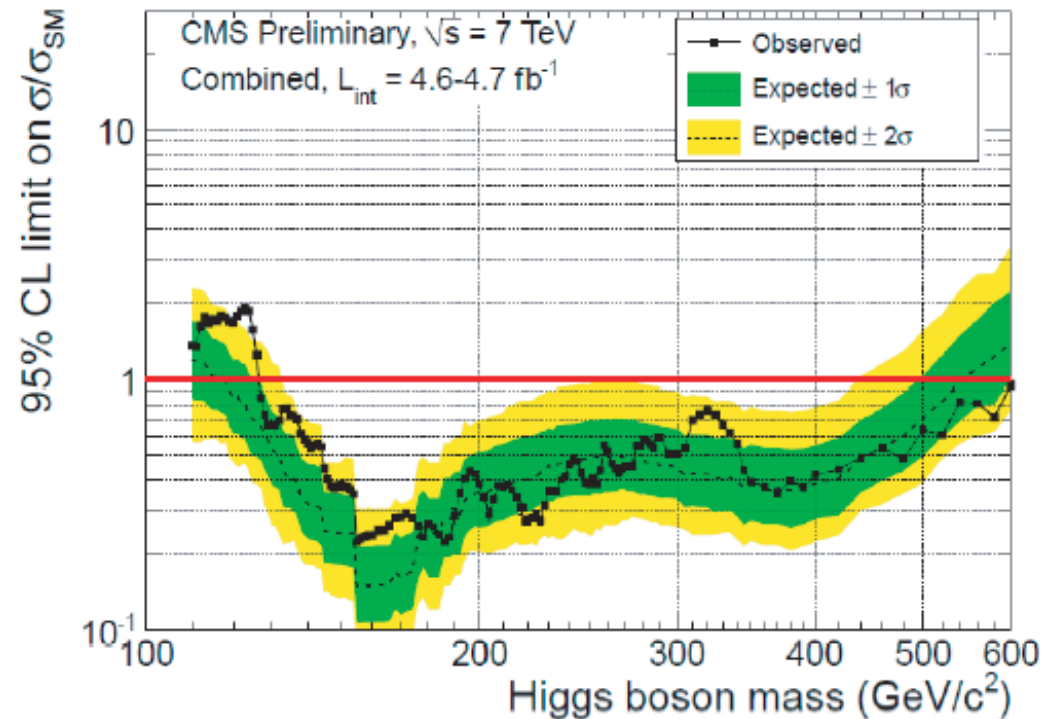
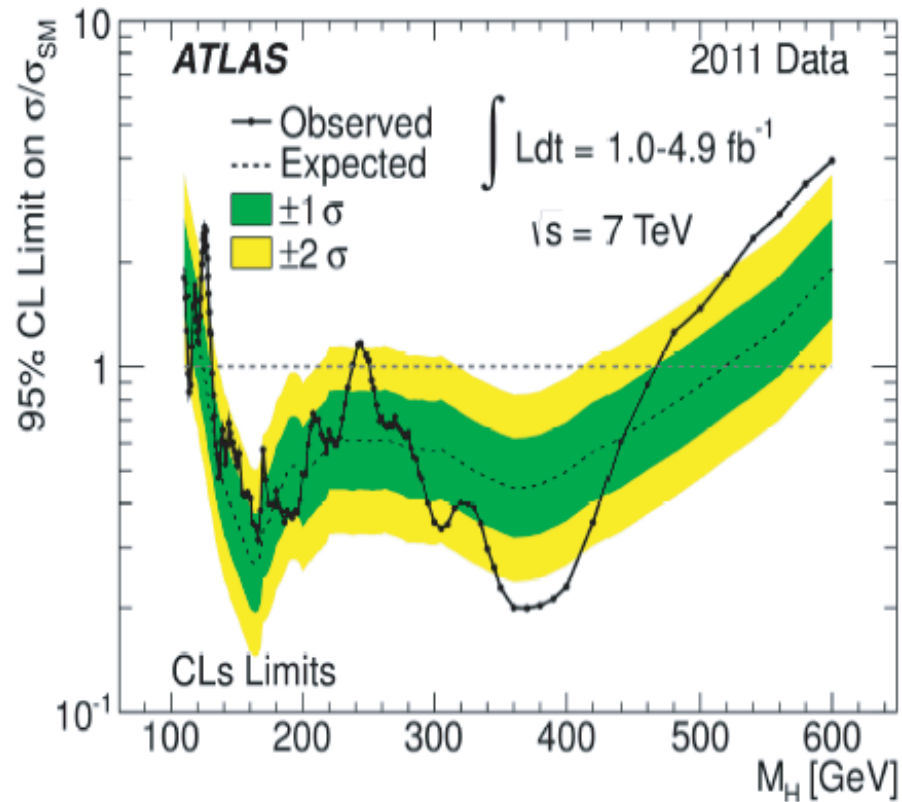
ATLAS @ LHC  
PLB 710 49 (2012)

# hints of 245 GeV Higgs bosons in earlier experiments

\* ATLAS-CONF-2011-135 August 21, 2011

Update of the Combination of Higgs Boson Searches

"The second was peaking at a Higgs boson mass hypothesis of 245 GeV"



Excess of events in  $H \rightarrow \gamma\gamma, H \rightarrow ZZ$

ATLAS near  $M_H \sim 126 \text{ GeV}$   
and near  $M_H \sim 245 \text{ GeV}$

CMS near  $M_H \sim 124 \text{ GeV}$   
and near  $M_H \sim 119.5 \text{ GeV}$

# excitation of Higgs boson by quench

dynamics of vacuum energy  $\Lambda$   
in cosmology

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

$$\omega \sim M_{\text{inflaton}}$$

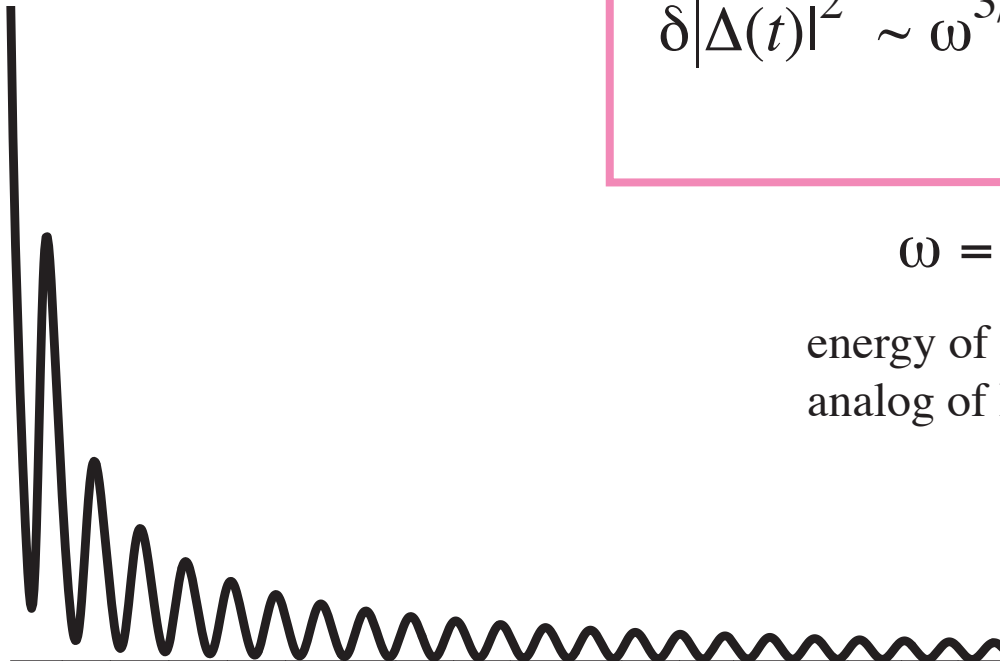
mass of Higgs inflation

dynamics of gap  $\Delta$   
in superconductor

$$\delta|\Delta(t)|^2 \sim \omega^{3/2} \frac{\sin \omega t}{t^{1/2}}$$

$$\omega = 2\Delta$$

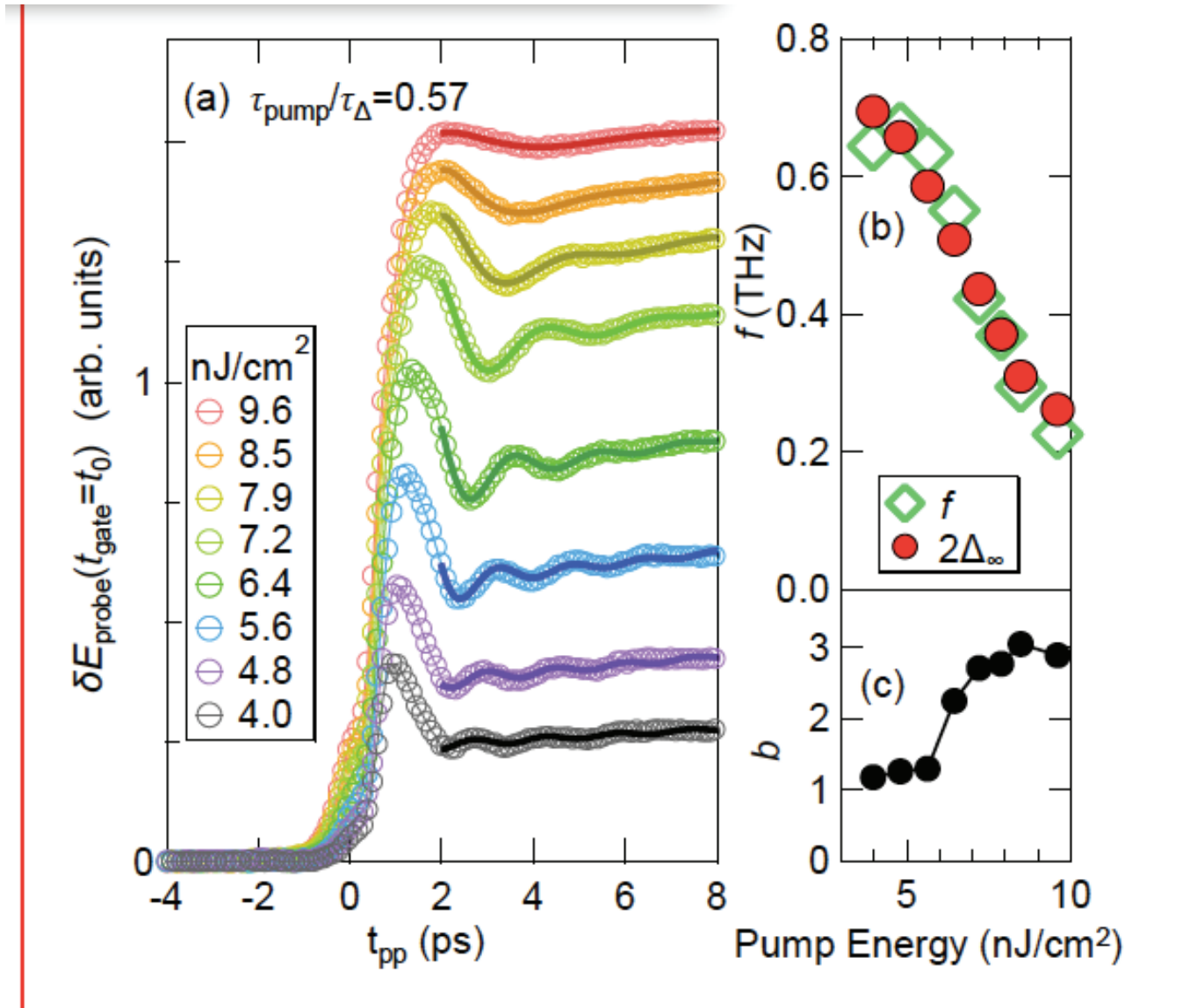
energy of amplitude mode:  
analog of Higgs mass



Starobinsky Higgs inflation

V. Gurarie, 0905.4498, 1307.1485, 1307.2256  
A.F. Volkov & S.M. Kogan, JETP **38**, 1018 (1974)  
Barankov & Levitov, ...

# Higgs amplitude mode after quench



Higgs Amplitude Mode in BCS Superconductors  
Nb<sub>1-x</sub>Ti<sub>x</sub> induced by Terahertz Pulse Excitation  
R. Matsunaga, et al. arXiv:1305.0381



# Majorana fermions in particle physics & cond-mat

## Majorana

(propagating fermions)



fermions in supersymmetric theories

Weyl “relativistic” fermions in bulk  $^3\text{He-A}$

Majorana edge states on surface of  $^3\text{He-B}$

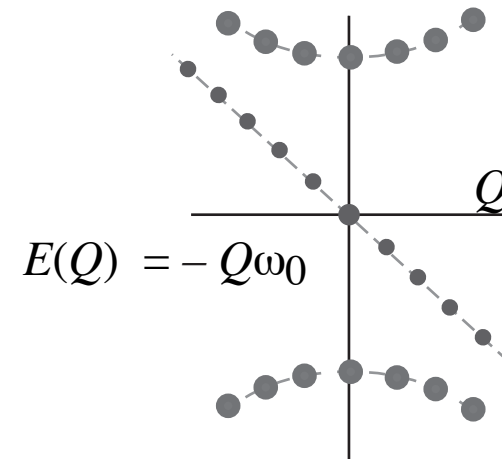
Majorana Valley: Fermi arc on surface of  $^3\text{He-A}$

## Majorino

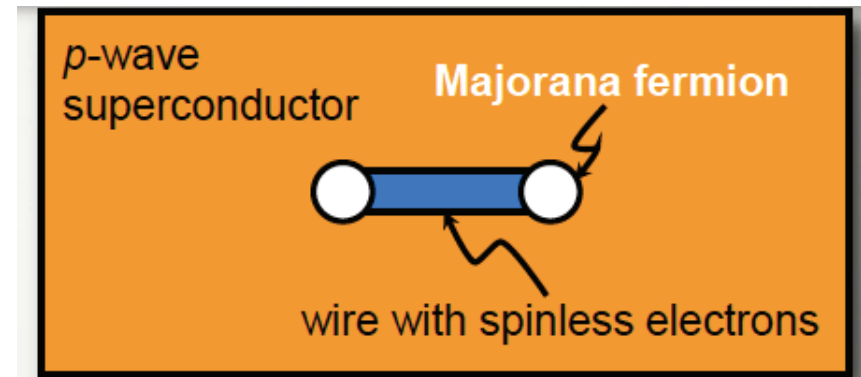
(zero-dimensional Majorana zero mode, Wilczek)

$$a^\dagger = a$$

zero mode in core of 2+1 p-wave superconductor

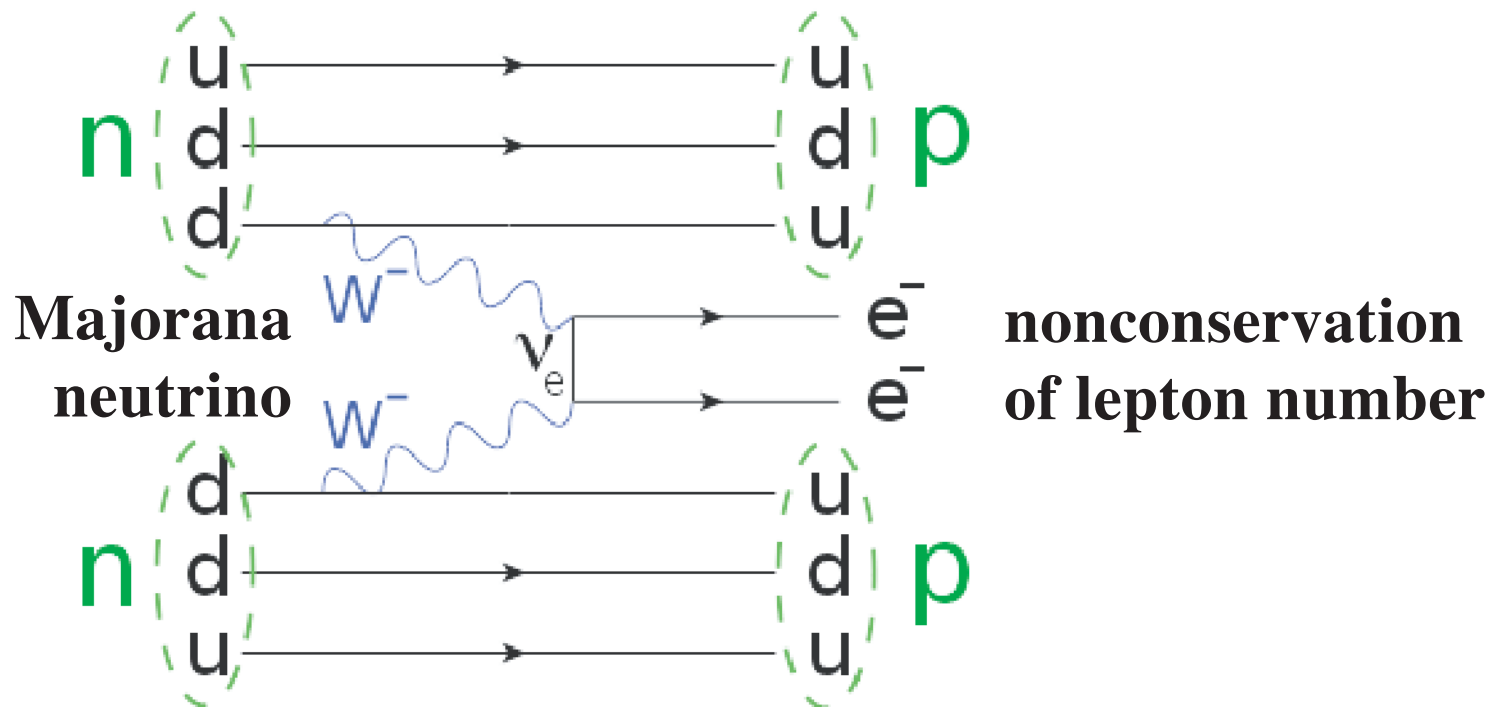


edge states in nanowires on superconductor

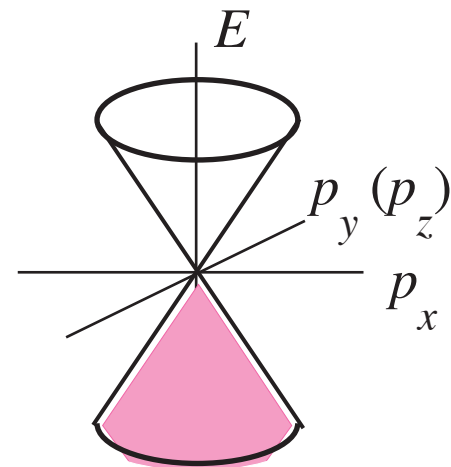
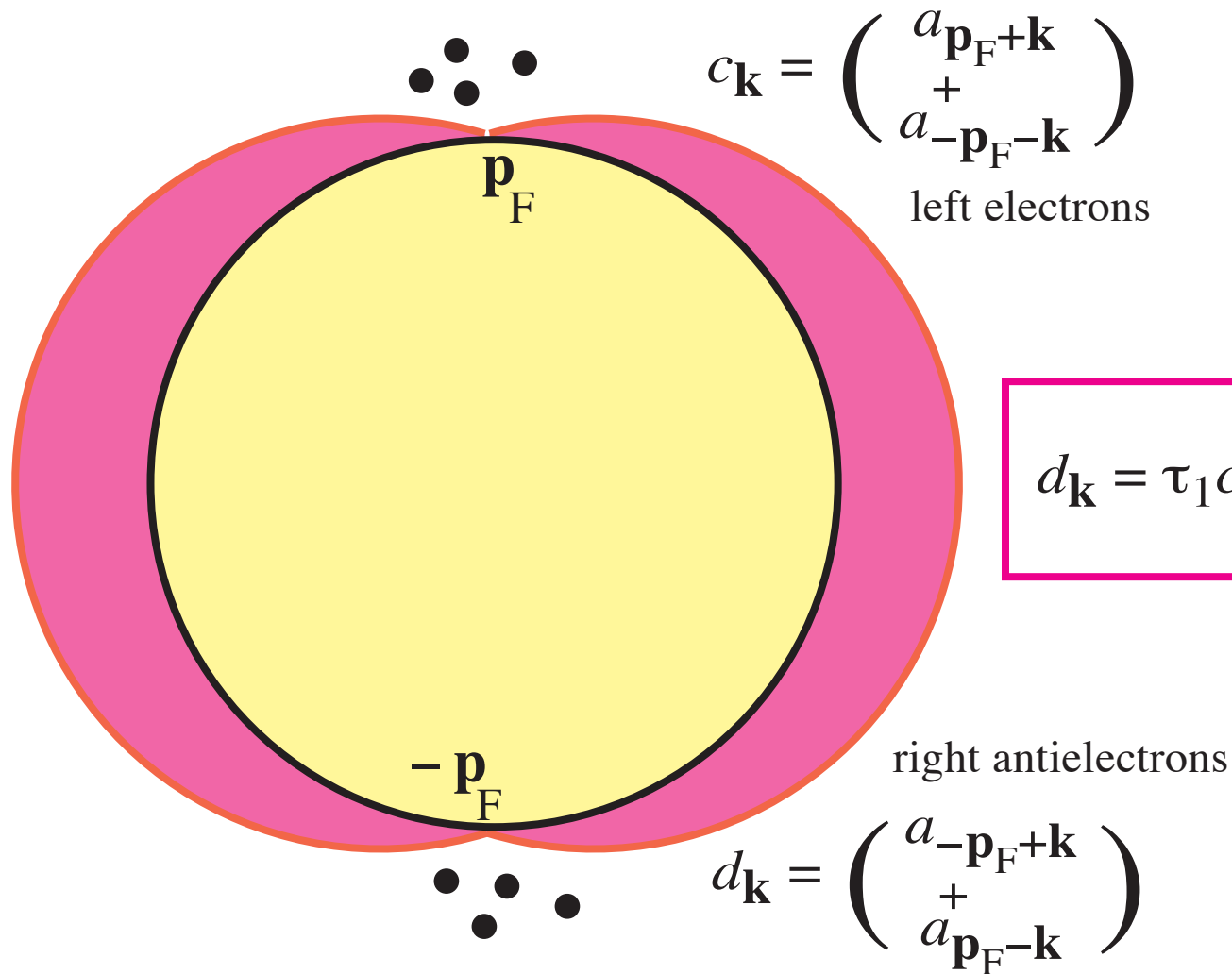


# neutrinoless double beta decay

observation of neutrinoless double beta decay  
necessarily implies Majorana nature of neutrinos

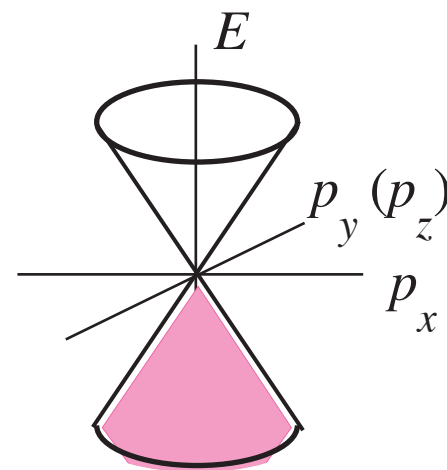


# Weyl fermions in $^3\text{He-A}$

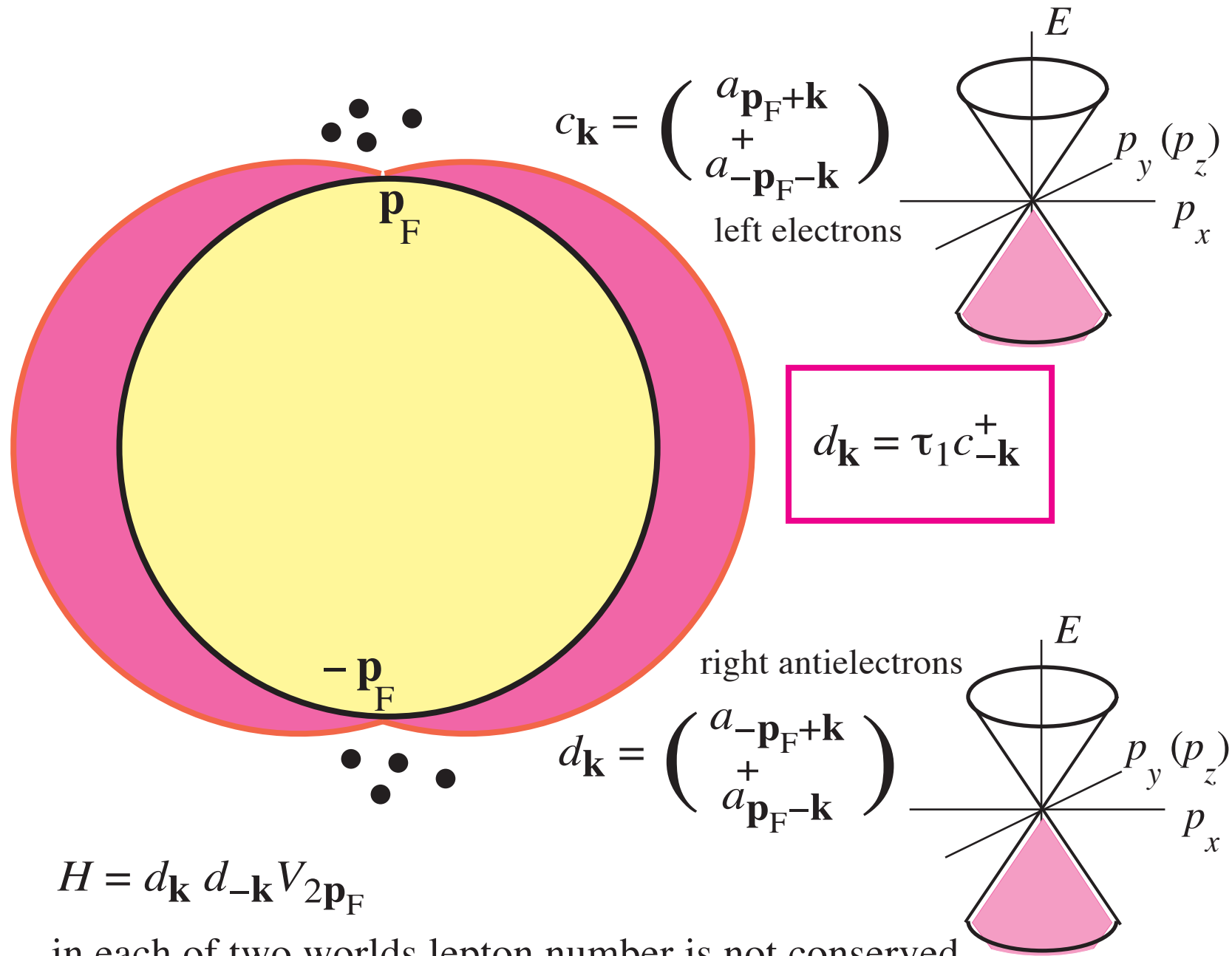


$$d_{\mathbf{k}} = \tau_1 c_{-\mathbf{k}}^+$$

**particles in two valleys  
are antiparticles**



maybe all particles in our world are Majorana ?



in each of two worlds lepton number is not conserved  
 neutrinoless double  $\beta$ -decay is possible  
 but only due to very short wavelength perturbation

## **conclusion**

- \* composite Higgs field is extension of Standard Model:  
from Ginzburg-Landau to BCS in Weyl semimetal**
- \* sum rule derived by Nambu for  $^3\text{He-B}$  can be extended to Standard Model**
- \* Nambu sum rule (if valid) may express masses of neutral & charged Higgs bosons  
via quark masses**
- \* more accurate analysis near 325 GeV and 245 GeV is needed**
- \* rigorous derivation of Nambu sum rule & its range of validity in fermionic systems:  
relativistic QFT, superconductors, magnetics, cold Fermi gases, graphene,...**