Structure of vortex lattices in rotating two-component Bose-Einstein condensates

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Abstract

We investigate numerically the structure of vortex lattices in rotating two-component Bose-Einstein condensates. The stationary solutions of the coupled Gross-Pitaevskii equations with a centrifugal term show various nontrivial patterns of vortex lattices, depending on the intra and interspecies interaction strengths g_{11}, g_{22} and g_{12}. For $\sqrt{g_{11}g_{22}} > g_{12}$ the vortices in each condensate form a vortex lattice, and one lattice is shifted from the other to reduce the overlap of the condensates. For $\sqrt{g_{11}g_{22}} < g_{12}$ when the phase separation of the condensate occurs, the vortex lattices are destroyed, forming the vortex sheet structure. The dynamics of vortex lattice formation is also discussed.

Key words: two-component Bose-Einstein condensate; quantized vortex lattice; rotation; phase separation

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1. Introduction

A quantized vortex lattice in a rotating trapped Bose-Einstein condensate (BEC) has been observed recently [1]. Although those experiments have been made with one-component BEC, rich phenomena are expected in rotating two-component BECs consisting of, for example, two different hyperfine spin states of atoms [2]. In this work, we study numerically static and dynamic properties of quantized vortices in two-component BECs under external rotation, by the coupled Gross-Pitaevskii equations

$$i\hbar\frac{\partial \Psi_i}{\partial t} = [h_i + g_{ii}|\Psi_i|^2 + g_{ij}|\Psi_j|^2]\Psi_i \quad i,j \in 1,2 \quad . \quad (1)$$

Here $h_i = -\hbar^2\nabla^2/2m_i + V_i^{\rm trap} - \Omega L_z$ with the atomic mass m_i, the harmonic trapping potential $V_i^{\rm trap} = m_i\omega_i^2 r^2/2$ and the centrifugal term $-\Omega L_z$. The intra- and interspecies atomic interactions are represented as $g_{ii} = 4\pi\hbar^2 a_{ii}/m_i$ and $g_{12} = 2\pi\hbar^2 a_{12}/m_{12}$ with the corresponding s-wave scattering lengths a_{ij} and the reduced mass m_{12}. The number of the parameter is reduced by the assumption $a_{11} = a_{22}$, $m_1 = m_2$ and $\omega_1 = \omega_2$. The numerical calculation is done in the two-dimensional space. The wave function is normalized as $\int dxdy|\Psi_i(x,y,t)|^2 = N_i^{2D}$, where the particle numbers per unit length along z-axis are set to be $N_1^{2D} = N_2^{2D} = N^{2D}$ for simplicity.

2. Vortex lattices in double condensates

First, we investigate the stationary state of the two-component BECs under rotation. The stationary solution is found by the imaginary time propagation of Eq. (1) from random initial wave functions. Figure 1 shows the density profile of the condensates with vortex lattices for $g_{12} = 0.8g_{11}$. The vortices in each condensate form a square lattice and the position of them in one condensate is shifted from the other. Because of the repulsive interaction between two condensates, the density of one condensate becomes high at the vor-

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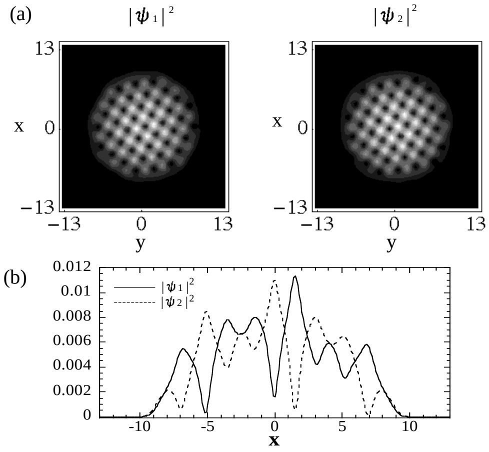

Fig. 1. (a) The density profile of the condensates for $\Omega = 0.7\omega_1$, $g_{12} = 0.8g_{11}$, and $g_{11}N^{2D} = 2000\hbar\omega_1$. (b) Cross section along the $y = 0$ line.

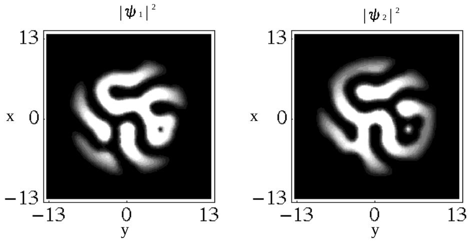

Fig. 2. The density profile of the condensates for $\Omega = 0.7\omega_1$, $g_{12} = 1.1g_{11}$, and $g_{11}N^{2D} = 2000\hbar\omega_1$.

tex cores of the other. Our result for $g_{12} < \sqrt{g_{11}g_{22}}$ is consistent with the results of Mueller and Ho [3], who studied analytically the vortex lattice configuration by minimizing the energy functional.

As g_{12} increases, the regular lattice structure is destroyed as shown in Fig. 2. For $g_{12} > \sqrt{g_{11}g_{22}}$ the two condensates almost separate from each other. Then vortices in one condensate are absorbed into the high-density region of the other, forming the vortex sheet structure [4]. The different initial conditions of the simulation yield various metastable states of the vortex sheet.

3. Dynamics of vortex lattice formation

We studied the dynamics of vortex lattice formation in one-component BEC [5]; the laser-induced stirring potential $V_{\mathrm{stir}} = (m\omega^2/2)(\epsilon_x x^2 + \epsilon_y y^2)$ excites the quadrupole surface mode [6] and its dynamical instability gives rise to the vortex generation [7]. Similar calculation is made for two-component BECs with $g_{12} < \sqrt{g_{11}g_{22}}$. Because of the interspecies interaction $g_{12} > 0$, the quadrupole mode has lower-lying and higher-lying mode, corresponding to the out-of-phase and in-phase motion [8], respectively. The typical dynamics of the vortex lattice formation via two different quadrupole modes is shown in Fig. 3. Which mode is excited is controlled by the trapping anisotropy ϵ_x and ϵ_y. Note that two kinds of dynamics occur at the same rotation frequency. These modes have the different excitation frequencies, which will affect the critical frequency for vortex generation. More detailed studies are in progress.

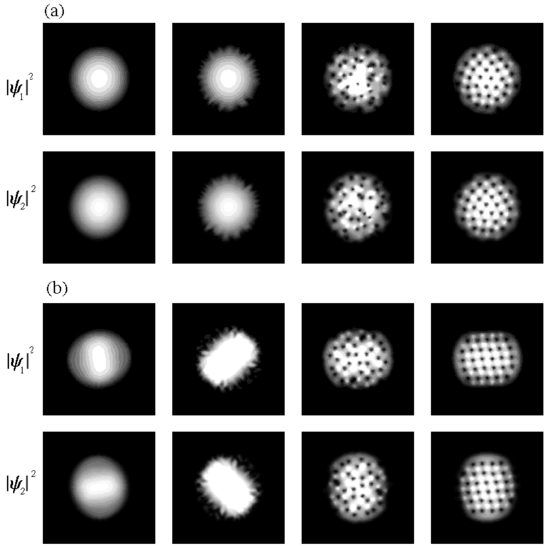

Fig. 3. Dynamics of vortex lattice formation in two-component BEC via (a) in-phase and (b) out-phase quadrupole mode. The stationary state in a non-rotating trap is used as the initial state. The in-phase mode is excited by implying $\epsilon_x > \epsilon_y$ for both ψ_1 and ψ_2, while the out-phase mode $\epsilon_x > \epsilon_y$ for ψ_1 and $\epsilon_x < \epsilon_y$ for ψ_2. The parameter is given as $\Omega = 0.6\omega_1$, $g_{12} = 0.8g_{11}$, and $g_{11}N^{2D} = 2000\hbar\omega_1$.

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[8] The out-of-phase motion means that the elongated direction of one condensate is perpendicular to that of the other. They are in line for the in-phase motion.