Quantum Turbulence

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Abstract

The paper is concerned with turbulence in a superfluid, in which flow is strongly influenced by the quantum effects that give rise to two-fluid behaviour, frictionless flow of the superfluid component, and restrictions on rotational motion. There is now significant experimental evidence relating to simple forms of turbulent flow in superfluid ^{4}He in circumstances where comparison is possible with an analogous flow in a classical fluid. The similarities and differences are analyzed, and the evidence is assessed for quasi-classical behaviour at large length scales in the quantum case. Dissipative processes at small length scales in the superfluid are discussed; they are shown to lead to an effective viscosity but to be based on novel quantum processes. The need for further experiments, especially at very low temperatures and in superfluid ^{3}He-B, is emphasized.

Key words: Turbulence. Superfluid. Vortex lines.

1. Introduction: turbulence in a classical fluid

Turbulence in classical fluids has long been an active and important field of study. Turbulence in a superfluid (quantum turbulence) has also been studied for many years, but only, for the most part, in connection with counterflow of the normal and superfluid components, where there is no classical analogue. It is only rather recently that types of turbulent superflow have been seriously studied for which a classical analogue does exist. This paper is concerned with such types and with what turn out to be interesting and instructive comparisons between the classical and quantum cases. The paper contains only an outline of the issues involved; details, including references to original papers, can be found in [1], along with an acknowledgement of helpful discussions with many friends and colleagues.

Classical turbulence occurs when the appropriate Reynolds number for the flow is large enough, the Reynolds number being a measure that characterizes the ratio of the non-linear inertial term in the Navier-Stokes equation to the dissipative viscous term. Classical turbulence is most easily treated in flows that are well-removed from any boundaries (free turbulence), especially in cases where the turbulence is homogeneous and isotropic. This latter form of turbulence can be produced by uniform flow through a grid, at a distance from the grid that is large compared with the grid mesh, M. The grid tends to produce turbulence on the scale of M, but the inertial term in the Navier-Stokes equation couples motion on different length scales and so causes energy to flow from the scale M to other scales, both larger and smaller, energy being conserved as long as the Reynolds number appropriate to the scale remains large compared with unity. Flow to larger length scales saturates at the scale of the channel that contains the flow. Flow to smaller scales continues, conserving energy, until the Reynolds number reaches a value of order unity, when there is dissipation by viscosity. The range of scale over which energy is conserved is called the *inertial range*. The turbulence can be described by various statistical quantities, of which the simplest is the *energy spectrum*, $E(k)$, where $E(k)dk$ is the turbulent energy associated with wavenumbers in the range dk, and where we use a Fourier analysis of the velocity field,

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wavenumber k being associated with scale k^{-1}. In the inertial range of homogeneous, isotropic, turbulence, in which energy is flowing towards high wavenumbers at a rate ϵ per unit mass, the energy spectrum takes the *Kolmogorov form*, at least approximately, given by

$$E(k) = C\epsilon^{2/3}k^{-5/3}, \tag{1}$$

where C is a dimensionless constant equal to about 1.5. This form of spectrum can be justified on dimensional grounds if it is assumed that within a wide inertial range the flow of energy in k-space takes place in a cascade, coupling being strong between motion with neighbouring wavevectors, so that the details of the way in which energy is fed into the cascade at small wavenumbers and lost by viscosity at large wavevectors is unimportant over most of the inertial range. When the Reynolds number of the flow near a particular wavenumber becomes of order unity, there is dissipation by viscosity, and the inertial range is terminated. It can be shown that the rate of dissipation of energy per unit mass is given by

$$\epsilon = \nu\langle\omega^2\rangle, \tag{2}$$

where ν is the kinematic viscosity of the fluid and $\langle\omega^2\rangle$ is the mean square vorticity in the fluid (most of the vorticity is concentrated near the cut off).

2. Turbulence in a superfluid

In the context of turbulence, a superfluid, such as that formed from ^{4}He, differs from a classical fluid in three respects: there is two-fluid behaviour; the superfluid component can flow without friction; and rotational flow of the superfluid is restricted to quantized line vortices, each carrying a single quantum of circulation, $\kappa = h/m_4$. When a vortex line moves relative to the normal fluid its core experiences a frictional force, commonly called mutual friction. We can ask how these differences affect turbulent flow, remembering that such flow is necessarily rotational. We consider only ^{4}He until section 6.

There are cases where quantum turbulence must be quite different from any type of classical turbulence, because it depends on the existence of two velocity fields. The clearest example is provided by counterflow turbulence, which is set up in superfluid ^{4}He when it carries a heat current, and which is actually *maintained* by a relative velocity of the two fluids. In other cases, however, quantum turbulence might have close classical analogues, especially if the two fluids flow with the same velocity or if the temperature is so low that the normal fluid is effectively absent. Although it has been suspected for many years that these other cases might exist, it is only within the past few years that any examples have been investigated in detail, experimentally and theoretically. In the rest of this paper we shall first discuss the two best and simplest examples, focussing on possible important general conclusions, and then go on to consider the potentially interesting case of very low temperatures, which has not yet been the subject of detailed experimental study.

3. Quasi-classical behaviour on large length scales

Since turbulence must involve rotational motion, turbulence in the superfluid component must take the form of an irregular tangle of quantized vortex lines. On length scales of order or less than the vortex-line spacing, ℓ, the superfluid flow field must be very different from that in any form of classical turbulence, but on larger length scales the vortex lines can be arranged to mimic classical flow patterns, the simplest example being the uniform array of parallel lines that allows the superfluid component to rotate with a containing vessel.

Perhaps the clearest experimental evidence comes from an experiment by Maurer and Tabeling. They generated turbulent flow in helium with two, four-bladed, counter-rotating discs, and they studied the frequency-spectrum of fluctuations in the fluid pressure at a point well away from the discs with a "total-head pressure tube". For a conventional fluid the spectrum so observed is simply related to the energy spectrum $E(k)$. The results were remarkable. There was no observable change in behaviour between the normal and superfluid phases, down to the lowest temperature studied, 1.4K. Furthermore, the spectrum was essentially of the Kolmogorov form (in the region of the turbulence near the pressure sensor the turbulence must therefore have been approximately homogeneous and isotropic). The pressure sensor had a size that was significantly larger than ℓ, so this *quasi-classical behaviour* related to large length scales. The only straightforward interpretation is that on these scales the two fluids have the same classical turbulent velocity fields. There are two possible reasons. According to the first, the velocity field of the superfluid component can not only mimic large-scale classical motion at any particular instant of time, but must also evolve in time according to classical equations, without any intervention from the normal fluid; mutual friction ensures simply that two velocity fields remain accurately locked together. According to the second, the dynamcs of the superfluid component is not inherently classical on large length scales, but classical behaviour is forced on it by the normal fluid through the effect of mutual friction. In view of the observation that quasi-classical behaviour exists for normal fluid fractions ranging from unity to values (∼0.07) much smaller than the superfluid fraction, it seems unlikely that the second view is tenable. However, final experimental confirmation must await experiments at much lower temperatures, as described later. In principle, computer simulations of superfluid turbulence at zero temperature could settle the issue; those carried out so far suggest that there can be quasi-classical behaviour, but they are not able to incorporate a fully adequate range of length scales.

4. Grid turbulence in a superfluid

The simplest type of classical turbulence is homogeneous and isotropic, and can be produced by steady flow through a grid. Experiments on *superfluid* grid turbulence above 1K have recently been reported by the Oregon group, with results that we now describe and discuss. The observational tool was the attenuation of second sound by the vortex lines in a particular small region, as a function of time after a grid had been towed through the helium. The measurement yields the time-dependence of the line density in the small region. This is determined in part by the behaviour of the turbulence on a scale larger than ℓ, and in part by dissipative processes occurring on the scale ℓ or less. No measurements have yet been attempted of the spectrum of pressure fluctuations, so there is as yet no *direct* evidence for quasi-classical behaviour in this case. However, an extension of the quasi-classical model, incorporating dissipation on small length scales, does account for the experimental results, thus providing evidence in favour of the quasi-classical model and about dissipative processes. It can be argued that these processes, due in part to the viscosity of the normal fluid and in part to mutual friction, act on length scales of order ℓ, mutual friction causing dissipation because on these scales the velocity fields of the two fluids cannot be the same. Thus, according to the extended quasi-classical model, there is an inertial range in which the two fluids have essentially the same velocity fields extending from length scales comparable with the grid mesh or the channel width to those of order ℓ. To obtain agreement with experiment the dissipation per unit mass, taking place on the scale ℓ, is given by

$$\epsilon = \nu' \, (\kappa L)^2 \,, \tag{3}$$

where L is the length of vortex line per unit volume, equal to ℓ^{-2}. The quantity $(\kappa L)^2$ can be interpreted as an effective mean square vorticity in the superfluid component (due to a more or less random array of vortex lines), so that Eq. (3) has a form similar to Eq. (2), ν' being an effective kinematic viscosity. Thus there seems in some sense to be quasi-classical behaviour not only on scales larger than ℓ, but also in the form taken by the dissipation on scales of order ℓ. Values of ν', as a function of temperature, can be obtained from the experimental results; putting $\nu' = \eta'/\rho$, where ρ is the total helium density, we find that η' is of order the viscosity of the normal fluid but with a quite different temperature dependence.

The form of Eq. (3) is closely related to an equation that describes the decay of a random vortex tangle in counterflow turbulence, namely

$$\frac{dL}{dt} = -\chi_2 \frac{\kappa}{2\pi} L^2, \tag{4}$$

where χ_2 is a temperature-dependent dimensionless parameter of order unity. Values of χ_2 were obtained by Schwarz in his pioneering simulations of counterflow turbulence for the case where the normal fluid is not turbulent. Very recently [1] it has been argued that Eq. (4) should indeed apply also in grid turbulence, and a simple model has been shown to lead to values of ν' in agreement with experiment, at least at temperatures below about 1.9K.

Above 1K Eq. (4) has the following physical basis. For a random vortex tangle the only characteristic velocity relating to the motion of the vortex lines is $u = \kappa/\ell$, if we ignore effects due to curvature of the lines that introduce only logarithmic corrections. If the normal fluid is at rest, this motion leads to a dissipation by mutual friction equal to $\gamma u^2 L$ per unit volume, where γ is a friction constant. It can be shown that this leads to the form of Eq. (4), with χ_2 related to the dimensionless parameter $\gamma/\rho_s\kappa$, where ρ_s is the density of the superfluid component. To obtain Eq. (3) we must take account also of viscous dissipation in the normal fluid, but this has the effect only of modifying the value of ν'. We see that, although Eq. (3) has a quasi-classical form, the physics of the dissipation is different and more complex than in a classical fluid, involving as it does both viscous dissipation in the normal fluid and the frictional interaction between vortex lines and the normal fluid. This difference has its origin in the fact that on the length scale ℓ at which dissipation occurs the turbulent flow of the superfluid must be very different from that of a classical fluid.

The analysis of grid turbulence that we have outlined provides further evidence in favour of quasi-classical behaviour on length scales larger than ℓ. But the experiments still relate to temperatures above 1K, where the normal fluid might be playing a role in forcing quasi-classical behaviour on the superfluid component. We turn now to the behaviour of ^{4}He at temperatures well below 1K, where the normal fluid has effectively disappeared, and where therefore we can study a particularly "pure" form of quantum turbulence.

5. Quantum turbulence at very low temperatures

This is potentially very interesting for two reasons: the behaviour of a turbulent superfluid component can be observed in the complete absence of normal fluid, so that arguments about quasi-classical behaviour being imposed by the normal fluid can be settled; and the absence of both mutual friction and any normal-fluid viscous dissipation leaves us with the problem of finding another dissipative process by which the turbulent energy can be lost. Experiments pose challenges because a study with second sound of the decay of vortex lines is no longer possible. The only relevant experiment to date is one at Lancaster, in which turbulence was generated with an vibrating grid and detected by the trapping of ions on the vortex lines. This experiment shows that turbulence can be generated with a moving grid, and that it does decay at a rate not very different from that at higher temperatures, but further interpretation has not yet proved possible. There is an urgent need to develop new experiments and techniques, and simplicity of interpretation points to experiments in which turbulence is produced by steady flow through a grid.

This almost complete lack of experimental evidence allows us to indulge unhindered in theoretical speculation. It is our own belief that the experiments will ultimately demonstrate quasi-classical behaviour in the pure superfluid in an inertial range at large length scales, the presence of any normal fluid being unnecessary. This leaves open the mechanism for dissipation at a sufficiently high wavenumber. We know that any type of turbulent motion leads to the generation of sound, so that a promising mechanism is the direct conversion of the turbulent energy into phonons (or perhaps rotons). A vortex undergoing oscillatory motion can radiate phonons, but it can be shown that the radiation is extremely weak at frequencies (of order κ/ℓ^2) relevant to vortex motion in a smooth tangle. Much higher frequencies are required, and these can come about only if energy flows into vortex motion on a length scale much less than ℓ. It is known from simulations that the vortices in a tangle will undergo frequent reconnections (at a rate per unit volume of order $\kappa\ell^{-5}$), and it was pointed out by Svistunov that each reconnection will leave sharp kinks on the reconnecting vortices. These kinks can be decomposed into a superposition of harmonic Kelvin waves, covering a wide range of high wavenumbers and high frequencies. The reconnection process itself will lead to some phonon (and roton?) production, which can be shown, however, to be relatively weak. Kelvin waves with a high enough frequency can certainly radiate phonons very effectively, so theoretical development must focus on the rate at which energy can flow from the tangle at length scale ℓ to these high-frequency Kelvin waves. The precise mechanisms are controversial, but we can nevertheless make some significant general statements. We make the plausible assumption that each reconnection leads to the transfer of energy of order $e_v\ell$ to the Kelvin waves, where $e_v \sim \rho_s\kappa^2$ is the energy per unit length of vortex at a very low temperature. We use the fact that the rate of reconnection is of order $\kappa\ell^{-5}$ per unit volume. And we assume that transferred energy is ultimately dissipated into phonons. Then the rate of loss of energy is easily seen to be of order $\kappa^3\ell^{-4}$ per unit mass of helium. If the relationship $L = \ell^{-2}$ were to hold, this loss would have the same form as the quasi-classical expression, Eq. (3), with $\nu' = \kappa$. However, this relationship between L and ℓ is not now quite correct, because it fails to take account of the fact that the lines in the tangle are not smooth but are crinkled by the presence of the Kelvin waves; this leads to an increase in L, for given ℓ, so that $L = g\ell^{-2}$ where the factor g is greater than unity. If g is a constant, then Eq. (3) continues to hold, but with $\nu' = \kappa/g^2$. The details of the process by which energy flows from the scale ℓ to that corresponding to the wavelength at which Kelvin waves radiate phonons efficiently are contained in the factor g. Since κ has the same order of magnitude as ν' measured at temperatures above 1K, we see that the rate of dissipation of energy may indeed be rather similar at extremely low temperatures to its value above 1K, as seems to be observed by the Lancaster group. This picture has been confirmed, at least in part, in computer simulations by Tsubota *et al*, which are described in another paper at this Conference; but these simulations do not have the spatial resolution required to see the details of the phonon radiation, which must occur on a very small length scale. On the experimental side, much more detailed measurements must now be carried out at very low temperatures to check both the general principles underlying our thinking and the details. The details are of considerable interest, because they involve processes that are novel to superfluid physics.

6. Turbulence in superfluid ^{3}He-B

In the present context we can regard ^{3}He-B as differing from superfluid ^{4}He in two respects: the normal fluid is much more viscous; and the vortex core parameter (coherence length) is much larger. The substantial consequences are discussed in [1], and again there is a need for experiments at very low temperatures.

References

[1] W. F. Vinen and J. J. Niemela, *J. Low Temp. Physics*, in the press.