Superconductor-insulator transitions in the two-dimensional limit

A. M. Goldman [a,1],

[a] *School of Physics and Astronomy, University of Minnesota, 116 Church St. S.E., Minneapolis, MN 55455, USA*

Abstract

Superconductor-insulator (SI) transitions in ultra-thin metal films, tuned either by magnetic field or disorder, have attracted substantial attention over the last decade, because of the possibility that they are quantum phase transitions. The bosonic picture of SI transitions proposed is at best only in qualitative agreement, as recent measurements suggest behavior more complex than a direct SI transition.

Key words: quantum critical points; fluctuations; superconductors

1. Introduction

The investigation of superconductivity in disordered ultrathin films began 60 years ago with the work of Shal'nikov [1] and continues to play an important role in contemporary condensed matter physics. Because Cooper pairs are formed from time-reversed eigenstates that are not strongly affected by disorder, one would not expect nonmagnetic impurities to have a significant effect on superconductivity [2]. On the other hand if one increased disorder such that Anderson localization occurred, superconductivity would cease to exist, even if an attractive electron-electron interaction were present. A perturbative theoretical description, based on weakening of the screening of the Coulomb repulsion with increasing disorder predicts the superconducting transition temperature to be a decreasing function of the high temperature sheet resistance of the film, R, measured in the normal state [4,5]. This fermionic theory is consistent with experimental studies of sputtered thin films [3], working best when the level of disorder is not too large [6].

Because the onset of superconductivity is a continuous phase transition, there are order parameter fluctuations. As a consequence, in two dimensions (2D), the transition becomes a topological, or Kosterlitz-Thouless-Berezinski transition, with the ordered phase being characterized by quasi-long-range rather than long-range order [7]. In the limit of zero temperature, these fluctuations are quantum mechanical rather than classical. The charge transfer associated with quantum fluctuations may have a more important effect on the superconductivity than the above-described fermionic processes. In some approximation Cooper pairs may be viewed as bosons, and approaches to the description of the superconductor-insulator (SI) transition based on a bosonic picture including quantum fluctuations have been developed. It is within this framework that the quenching of superconductivity by disorder, or by magnetic field has been suggested to be a zero-temperature quantum phase transition (QPT) [8]. This transition would be one of the most fundamental of QPTs as it would arise from the uncertainty between phase and particle number in the superconductor.

This approach to the superconductor-insulator transition was motivated in part by the observation, using techniques described below, of a clear separation between superconducting and insulating behavior [9] in a sequence of films of different sheet resistances. This occurred at a sheet resistance very close to the quantum resistance for electron pairs, $h/4e^2$, or 6450Ω (See Fig. 1) and outside the realm in which the theory of Refs. [4] and [5]would be expected to apply. The set of curves of $R(T)$ taken together resemble renormaliza-

[1] E-mail: goldman@physics.umn.edu

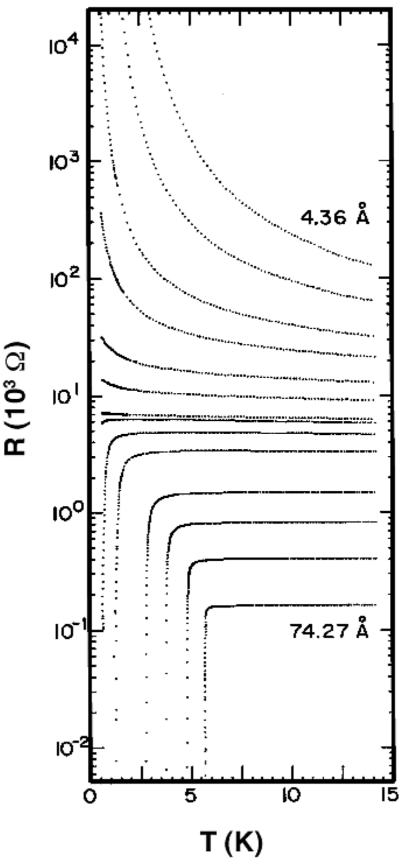

Fig. 1. Evolution of the temperature dependence of the resistance $R(T)$ with thickness of a series of a-Bi films deposited onto a-Ge. Fewer than half of the traces are shown. Adapted from Ref. [9]

tion flows to an unstable fixed point at zero temperature. This result was followed by the very important finding that the SI transition could be traversed by applying a magnetic field [10–12].

In this article we will focus on studies of SI transitions in quenched condensed films. Section 2 contains a discussion of the experimental approach. In Section 3, the results of finite-size scaling analyses supporting the existence of quantum critical points separating superconducting and insulating ground states are presented. Here film thickness and magnetic field are the tuning parameters. The final section considers a number of unresolved issues. These include persistent evidence of the role of fermionic degrees of freedom, and more recent concerns relating to the nature of the ground states, the presence of metallic phases, and the role of dissipation. Some of the experimental challenges that must be overcome in order to develop a complete understanding of this problem are also identified.

2. Experimental

The extent of modification of the properties of superconducting films by disorder depends on its strength and geometrical scale relative to other lengths such as the inverse Fermi momentum, the electronic mean free path, the London penetration depth, the BCS coherence length, and the zero-temperature Ginzburg-Landau coherence length. The length scale for disorder in a film is determined by processing. Without special precautions, thin films as deposited may be inhomogeneous, with a length scale for disorder that is mesoscopic, *i.e.*, $\sim 10^2 \mathring{A}$. In certain sputtered films, and in metal films deposited onto *a*-Ge substrates held at liquid helium temperatures [13], the scale for disorder can be at or near atomic lengths. Without an *a*-Ge underlayer, the set of curves of $R(T)$ as a function of thickness in such quench-deposited films is more complex than that of Fig. 1, with reentrant superconducting behavior and local superconductivity and metallic behavior preceding the appearance of global superconducting order [14]. Films with such complexity exhibit mesoscale disorder as demonstrated in atomic force microscope studies [15]. When the substrates are pre-coated with a thin layer of *a*-Ge, atomic scale disorder is obtained because the underlayer enhances wetting of the substrate by the film, preventing agglomeration into mesoscale clusters. A caveat is that there may still be clusters as structural studies are not definitive. The underlayer, although by itself not conducting, may enhance the electrical coupling between clusters giving rise to electrical connectivity at very early stages of growth.

Most of our experiments were carried out by repeated deposition of small increments of metals that are superconducting in bulk onto substrates held at liquid helium temperatures, alternated with *in situ* electrical measurements at dilution refrigerator temperatures. Figure 1 is an example of what can be obtained using this approach. Many of the other experiments on SI transitions employed sputtered films of In_xO_y, Mo_xSi_y, and Mo_xGe_y [10–12,16]. Structural characterizations of films of the same composition as those use to study the SI transition, indicated that they amorphous and free of clusters. High temperature superconductors in various configurations have also been investigated [17,18].

3. Scaling of the Thickness and Magnetic-Field Driven SI Transitions

The physics of dirty bosons [8] is concerned with the problem of Bose particles in a random medium. Although originally considered in the context of helium, it has been used to treat superconductors by considering Cooper pairs to be point-like, charge $2e$ bosons in a random potential, interacting with a long range Coulomb force. This has been justified because models of superconductivity based on a finite temperature Bose condensation and those based on the BCS theory belong to the same universality class. In the dirty boson picture, the SI transition tuned by disorder or magnetic field is a QPT [19].

A standard approach to establishing the existence of a QPT is to carry out a finite size scaling analysis of transport data. To appreciate how this works, we note a key feature of QPTs, the interplay of dynamics and thermodynamics. A *d-dimensional* quantum system at finite temperature is described in the $T \to 0$ limit, as long as the dynamical critical exponent $z = 1$, as a classical system of $d+1$ dimensions, with the finite extent of the system in the extra dimension being given by - $\hbar\beta$ in units of time, where $\beta = 1/k_B T$. This extra dimension is finite at nonzero temperature. More generally, when $z \neq 1$ space and time do not enter in the same fashion in the equivalent classical problem, which is then a $d + z$ dimensional problem.

Near the quantum critical point (QCP) there are divergent correlation lengths that are determined by the deviation from the critical point, written as $\delta = |K - K_c|$, where K is the control parameter (i.e., disorder, thickness, magnetic field, etc.) and K_c is its critical value. Then the spacial and temporal correlation lengths are written as $\xi \sim |\delta|^{-\nu}$, and $\xi_t \sim \xi^z$, where the correlation length exponent is ν and the dynamical critical exponent is z. Physical quantities in the critical region are homogeneous functions of the independent variables in the problem. As mentioned, the effect of considering $T > 0$ in the statistical mechanics is to force the "temporal" dimension of the problem to be finite. The resistance of a 2D system in the critical regime then follows the finite-size scaling relation

$$R(\delta, T) = R_c f(\delta T^{-1/\nu z}) \tag{1}$$

where $\delta = |d - d_c|$ or $\delta = |B - B_c|$ for either the thickness or the magnetic-field-tuned transitions. One can carry out a similar analysis of the nonlinear dependence of resistance on electric field, writing

$$R(\delta, E) = R_c f(\delta E^{-1/\nu(z+1)}) \tag{2}$$

where E is the electric field across the sample. The collapse of data for both the field- and thickness-tuned transitions is shown in Fig. 2 [20]. The critical exponent product for the former is $\nu z = 1.2 \pm 0.2$. For the latter its is $\nu z = 0.7 \pm 0.2$, independent of film thickness. For amorphous films of In_xO_y and Mo_xGe_y νz was found to be 1.3.

Analysis of data on the thickness-driven transition in a magnetic field yielded the exponent product $\nu z = 1.4 \pm 0.2$, independent of magnetic field. This is close

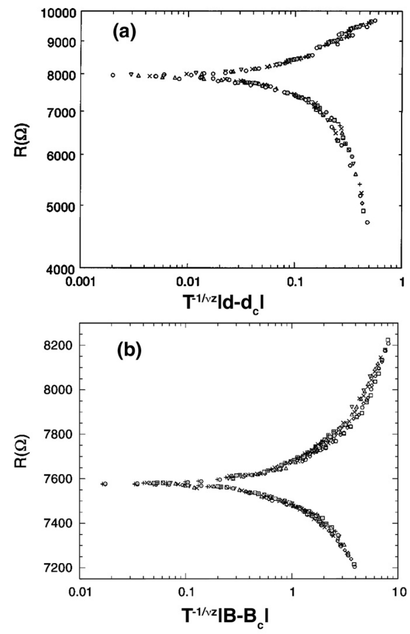

Fig. 2. a) Finite size scaling analysis of data of resistance vs. thickness for a sequence of a-Bi films grown on a-Ge. Different symbols represent different temperatures ranging upwards from 0.15K; b) Finite size scaling analysis of resistance vs. magnetic field for the same temperature range. The film was initially superconducting. (Adapted from Ref. [20].)

to the value obtained for the zero-field transition carried out on the same set of films. For the electric field driven transition (not shown), the best fit obtained corresponds to $\nu(z + 1)$=1.4. Combining these with the result of scaling the field-tuned data, one finds $z = 1$ and $\nu = 0.7$. This value of z is the same as was found for Mo_xGe_y films [11].

Our result, $\nu = 0.7$, is inconsistent with an "exact" theorem, which predicts $\nu \geq 1$ in two dimensions in a presence of disorder [21]. It has been suggested that disorder averaging may introduce a new correlation length, different from the intrinsic one, which might lead to $\nu < 1$ even for a disordered system [22]. A value of $\nu = 0.7$ actually corresponds to the universality class of the classical 3D XY model, which would be appropriate in the case if there were no disorder. Numerical simulations of *(2+1)-dimensional* XY and Boson-Hubbard models without disorder, find $z = 1$ and $\nu = 0.7$. In recent work on *a*-Be, it was reported that $\nu z = 0.7$ was found when the measurements were carried out at high current densities, and 1.2 at low currents [23]. We investigated the current-dependence of the resistance of our films and our current densities were comparable to, or lower, than those reported in that work. As a consequence this may not be related

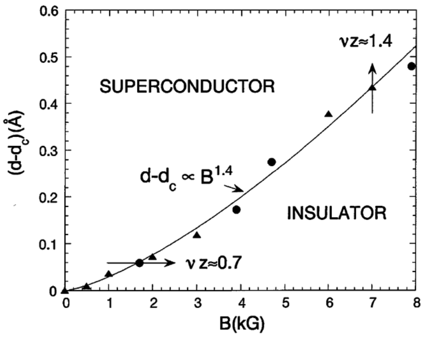

Fig. 3. Nominal phase diagram in the d-B plane in the zero temperature limit. The points on the boundary from magnetic field tuned transitions are circles whereas those for disorder-tuned transitions are triangles. Values of the exponent products are shown next to the arrows which delineate the manner in which the boundary was crossed. Here $d_c$is the zero-field critical thickness. (Adapted from Ref. [20].)

to our observations.

The various data for the critical thickness and fields of the above transitions can be combined to yield a "phase boundary" between films which are "superconducting" ($dR/dT < 0, T > 0$), and "insulating" ($dR/dT < 0, T > 0$). The boundary satisfies a power law of the form $B_c \sim |d - d_c|^x$, where $x = 0.7$, as determined from Fig. 3. This exponent leads to an inconsistency in that along the boundary one might expect $B_c \sim |d - d_c|^{2\nu}$ [8] which would imply that $\nu = 0.35$. The critical exponent product also depends on whether the boundary is crossed vertically or horizontally in the thickness-magnetic field plane. If the line were a true phase boundary the exponents would have to be the same.

An exponent product νz ranging from 1.2 to 1.4 is very close to the exponent for percolation in 2D. It is possible that the thickness-tuned transition in our work is percolative, whereas the field-tuned one is not. There is a major difference between magnetic field- and thickness-tuned transitions: when the transition is tuned by the magnetic field, the microstructure of the sample stays fixed, while in the case of the thickness tuned transitions it changes slightly with each film in the sequence. It may be that in this case the percolation effects become relevant, complicating the determination of the critical exponents [24]. There are theoretical models [25] in which disorder brings about intrinsic inhomogeneity of the order parameter. This could lead to percolative effects in the thickness-tuned transition.

4. Discussion

There are a number of microscopic features of these 2D systems that are not treated in dirty boson models. Tunneling studies, which determine the density of states, suggest that bosons cannot be the full story [26]. For both thickness- and field-tuned SI transitions the energy gap is found to scale with the transition temperature and to disappear in the insulating state. These results imply that amplitude as well as phase fluctuations are associated with the SI transition. If the order parameter were to vanish in the insulating state, as one might infer from the vanishing of the energy gap, then the dirty boson picture with local superconducting order in the insulating phase might be totally irrelevant. Such a conclusion should be treated with care as the vanishing of the gap feature in the tunneling conductance could result from other effects such as pair breaking by phase fluctuations. Tunneling studies of the insulating state might also be emphasizing regions of the samples containing quasi-localized single electron states below the gap, or those in which the amplitude fluctuations break the system into superconducting "islands" with finite spectral gaps in the density of states, as predicted Ghosal *et al.* [25,27]. This problem could be clarified by spatially resolved scanning tunneling spectroscopy at low temperatures, as it should be possible detect local variations in the density of states. At any rate, tunneling experiments raise serious questions as to the completeness of a phase-only picture of the transition.

A major prediction of the dirty boson model is that there is a universal limiting resistance at the QCP for both field- and disorder-tuned transitions [8]. This is not found [28]. The spread of limiting resistances may be extrinsic as there are morphological differences between films of different materials made by different processes. In particular films claimed to be homogeneous may contain grains. The Josephson coupling between grains in such films will be determined by the ratio of the electrostatic energy to the Josephson coupling energy which will depend on geometry. This could lead to a geometry-dependent critical resistance. Alternatively the variations may be a consequence of material-specific features such as the strength of strong spin-orbit coupling. Finally the data used to analyze the transition may not be from the critical regime of the QCP. The size of this regime is not known, and may require studies at lower temperatures with values of the tuning parameter closer to critical. There has been a suggestion that local dissipation due to gapless electronic excitations might change the universality class of the system and lead to a non-universal critical resistance [29]. In this picture, the critical resistance is predicted to increase with increased damping due to dissipation, which might be expected to increase with decreasing normal state resistance.

There are features of various experiments that support the dirty boson picture. For instance, Cooper pairs are predicted to be present in the insulator. There is evidence of this in Hall effect studies of the insulator [30], which suggest a crossover between two distinct insulating phases. When the longitudinal resistance, R_{xx}, and the transverse Hall resistance, R_{xy}, are measured on the same film, a divergence of R_{xx} is found at a lower field than that at which R_{xy} begins to diverge. With increasing B there is first a transition to one insulating phase, and then a crossover or a transition to a second phase at a higher field. At the high field feature there is also a drop in the magnetoresistance. This scenario could be consistent with a picture in which the first transition is between a superconductor and a Bose insulator, a state with nonzero pairing but which has infinite resistance at zero temperature. The second feature would be a crossover or a transition to an electronic insulator without pairing, i.e., to a Fermi insulator.

A second piece of evidence supporting the dirty boson picture is in the magnetoresistance of quench-deposited films [31]. Just on the insulating side of the SI transition in a regime in which the resistivity goes as lnT, the magnetoresistance is positive, with the magnetic field applied perpendicular to the plane. The field-perpendicular-to-the-plane magnetoresistance is greater than that obtained with the field parallel to the plane. The difference between the two, which is linear in field, is a measure of the orbital magnetoresistance. This is what would be expected for flux flow of vortices.

The success of finite size scaling analyses of the superconductor-insulator transitions as a function of either thickness or magnetic field provides evidence for there being $T = 0$ QCPs. The question is whether the measurements that have been made in the range from 1.0 K down to 0.1 K accurately predict the behavior in the zero temperature limit [32] . Certainly measurements at lower temperatures would increase the confidence level in this regard. In recent studies of the magnetic field-tuned SI transition, Mason and Kapitulnik [33] demonstrated scaling at relatively high temperatures. However as the temperature was reduced further, the resistance saturated, scaling was disrupted and the system entered a metallic regime. At very low magnetic fields, at low temperatures, the resistance dropped by more than three orders of magnitude in some instances. The addition of a parallel ground plane in proximity to the film changed the character of the transition, by lowering the resistance and enhancing superconductivity. These observations were interpreted as evidence of a role for dissipation in the SI transition [34]. Dissipation was discussed some time ago in the context of explaining the SI transition in granular thin film systems and arrays of Josephson junctions [35], and is again under active discussion [36]. Other theoretical approaches to metallic phases have been presented recently by several groups [37–39]. There has been a recent proposal, in the context of the quantum rotor model to the effect that the metallic regime is a phase glass [40].

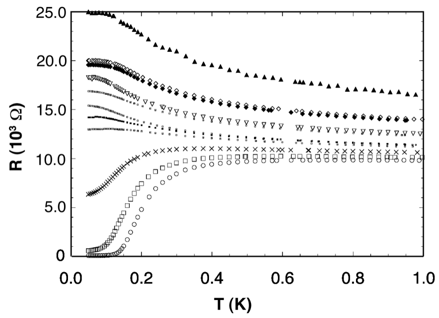

Fig. 4. Evolution of $R(T)$ of an a-Bi film, measured at temperatures down to 0.050K. The systematics of measurements in a magnetic field (not shown) suggest that the flattening is not a consequence of the electrons not cooling. Nominal film thicknesses from top to bottom are 8.5, 8.7, 8.8, 8.85, 8.91, 8.99, 9.05, 9.09, 9.19, 9.25, and 9.3Å. (Adapted from Ref. [41])

A metallic regime, which is quite evident in studies of ganular ultrathin films, has recently been observed in studies of *a*-Bi at temperatures down to 50 mK as shown in Fig. 4 [41]. In contrast with the work on granular films, the flattening occurs at much lower temperatures. It is also found in a range of films on *both* the insulating and superconducting sides of the transition. Much higher and lower resistance films are truly insulating and superconducting respectively. Scaling works only if high temperature data are considered. In fact the data above 150 mK for these films was scaled and the results are identical to what was reported earlier. We are reasonably certain that there was no transition to superconductivity down to much lower temperatures. Although the thermometer used in these experiments bottomed out at 50mK, other studies indicate that the minimum temperature was actually 20 mK. These measurements indicate that for an extended range of disorder, the ground state is a metal rather than a superconductor or an insulator.

There are experimental caveats that must be asserted at this point. The validation of a metallic ground state of 2D films will require more elaborate efforts to unequivocally establish that saturation is not a consequence of a failure to cool the electrons. The experimental systems must be carefully shielded from extrinsic noise and from thermal noise generated at temperatures above the limiting low temperature of the apparatus. The problem is exacerbated by the fact that the metal layers are less than 10 to 20Å in thickness. The determination of the true ground state of the system will only be made if this important experimental challenge is overcome.

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